Answer on Question #84610 – Math – Statistics and Probability

Question

The label on boxes of laundry detergent claims the boxes contain 39 ounces of detergent. There is, however, some variability in the filling process. The amount of laundry detergent per box is known to follow a normal distribution with standard deviation 0.6 ounces. A consumer will measure the contents of a random sample of 10 boxes and conduct a hypothesis test at the 10% level of significance to determine whether the true mean content of all boxes of laundry detergent is less than 39 ounces.

a. What is the power of the test if the true mean is actually 38.5 ounces?

b. What is the probability of making a Type II error if the true mean is actually 38.7 ounces?

Keep 4 decimal places in intermediate calculations and report your final answer to 4 decimal places.

Solution

We have:

H_0 =”the true mean content of boxes is equal 39 ounces”

H_1 =”the true mean content of boxes is less than 39 ounces”.

It is one-side alternative. We choose C such that

$$P_{H_0}\left(\bar{X} \le C\right) = 0.1$$

We have

$$P_{H_0}\left(\bar{X} \le C\right) = P_{H_0}\left(\frac{\bar{X}-39}{\frac{\sigma}{\sqrt{n}}} \le \frac{C-39}{\frac{\sigma}{\sqrt{n}}}\right) = F\left(\frac{C-39}{0.6}\cdot\sqrt{10}\right) =$$

$$= F\left(\frac{C-39}{0.1897}\right)$$

Then

$$\frac{C-39}{0.1897} = F^{-1}(0.1)$$

from which

$$C = 0.1897F^{-1}(0.1) + 39 = 0.1897\cdot(-1.2815) + 39 = 38.7565$$

Thus if $\bar{x} \le 38.7565$ the null hypothesis is rejected.

a. The power of the test is

$$P_{H_1}\left(reject\ H_0\right) = P_{a=38.5}\left(\bar{x} \le 38.7565\right) =$$

$$= P_{a=38.5}\left(\frac{\bar{x}-38.5}{\frac{0.6}{\sqrt{10}}} \le \frac{38.7565-38.5}{\frac{0.6}{\sqrt{10}}}\right) = F\left(\frac{38.7565-38.5}{\frac{0.6}{\sqrt{10}}}\right) =$$

$$= F\left(1.355\right) = 0.9122$$

b. The probability of making type 2 error is

$$P_{H_1}\left(accept\ H_0\right) = P_{a=38.7}\left(\bar{x} > 38.7565\right) =$$

$$= P_{a=38.7}\left(\frac{\bar{x}-38.7}{\frac{0.6}{\sqrt{10}}} > \frac{38.7565-38.7}{\frac{0.6}{\sqrt{10}}}\right) =$$

$$= 1 - F\left(\frac{38.7565-38.7}{\frac{0.6}{\sqrt{10}}}\right) = 1 - F\left(0.300\right) = 0.3821$$