Answer on Question #84553 – Math – Statistics and Probability

Question

Is the statement true or false?

In queueing theory, if the arrivals are according to Poisson distribution with parameter λ, the inter-arrival time is exponential with parameter e^{λ}.

Solution

Customers arrive according to a Poisson distribution with parameter λT, where λ, is the average number of arrivals per unit time

$$P(n \text{ arrivals in interval } T) = \frac{(\lambda T)^n e^{-\lambda T}}{n!},$$

$n = \text{number of arrivals in } T$

$E[n] = \lambda T$

$E[n^2] = \lambda T + (\lambda T)^2$

$\sigma^2 = E[(n - E[n])^2] = E[n^2] - (E[n])^2 = \lambda T$

Time that elapses between arrivals (IA)

$$P(IA \le t) = 1 - P(IA > t) = 1 - P(0 \text{ arrivals in time } t) = 1 - \frac{(\lambda T)^0 e^{-\lambda T}}{0!} =$$

$$= 1 - e^{-\lambda T}$$

This is known as the exponential distribution

$\text{Inter} - \text{arrival CDF } = F_{IA}(t) = 1 - e^{-\lambda T}$

$$\text{Inter} - \text{arrival PDF } = \frac{d}{dt}\left(F_{IA}(t)\right) = \lambda e^{-\lambda T}$$

The inter-arrival time has an exponential distribution with parameter λ.

The statement that the inter-arrival time is exponential with parameter e^{λ} is False.

Answer: The statement is False.