## Answer on Question #84483 – Math – Statistics and Probability

## Question

A population consists of three numbers 2, 5, 8. Enumerate all possible samples of size 2 which can be drawn without replacement from this population. Verify that the sample mean is an unbiased estimate of the population mean. Calculate the standard error of the sample mean.

## Solution

All possible samples of size 2 which can be drawn without replacement from this population:

$2, 5; 2, 8; 5, 8; 5,2; 8,2; 8, 5$

$$\overline{x}_1 = \frac{2+5}{2} = 3.5;\ \overline{x}_2 = \frac{2+8}{2} = 5;\ \overline{x}_3 = \frac{5+8}{2} = 6.5;$$

$$\overline{x}_4 = \frac{5+2}{2} = 3.5;\ \overline{x}_5 = \frac{8+2}{2} = 5;\ \overline{x}_6 = \frac{8+5}{2} = 6.5$$

$$\frac{\overline{x}_1 + \overline{x}_2 + \overline{x}_3 + \overline{x}_4 + \overline{x}_5 + \overline{x}_6}{6} = \frac{3.5 + 5 + 6.5 + 3.5 + 5 + 6.5}{6} = 5$$

$$\mu = \frac{2+5+8}{3} = 5$$

The mean of a sample is an unbiased estimate of the mean of the population from which the sample was drawn.

$$\frac{\overline{x}_1 + \overline{x}_2 + \overline{x}_3 + \overline{x}_4 + \overline{x}_5 + \overline{x}_6}{6} = 5 = \mu$$

$$\sigma^2 = \frac{1}{n} \cdot \sum_{i=1}^{n} (x_i - \mu)^2$$

$$\sigma^2 = \frac{1}{3} \cdot \sum_{i=1}^{3} (x_i - 5)^2 = \frac{(2-5)^2 + (5-5)^2 + (5-8)^2}{3} = 6$$

Calculate the standard error of the sample mean

$$\sigma_M = \frac{\sigma}{\sqrt{n}}$$

$$\sigma_M = \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}$$