Answer to Question #84333 – Math – Statistics and Probability

In a railway yard goods trains arrive at the rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes, calculate the following:

Question

(i) The average number of trains in the queue.

Solution

Mean arrival rate:

$$\lambda = \frac{30\ trains}{24\ hours} = 1.25\ \frac{trains}{hour}$$

Mean service rate:

$$\mu = \frac{1\ train}{36\ minutes} = \frac{1\ train}{0.6\ hours} = 1.67\ \frac{trains}{hour}$$

$$\rho = \frac{\lambda}{\mu} = \frac{1.25}{1.67} = 0.75$$

$$L = \frac{\rho}{1-\rho} = \frac{0.75}{1-0.75} = 3\ trains$$

Answer: $L = 3\ trains.$

Question

(ii) The probability that the queue size is greater than or equal to 10.

Solution

If the queue size is greater than or equal to 10 then the number of trains in a railway yard is greater than or equal to 10. The probability that there are i trains in a railway yard is equal to $(1-\rho)\rho^i$

Then the required probability is

$P(L \geq 10) = \sum_{i=10}^{\infty}(1-\rho)\rho^i = (1-0.75)\sum_{i=10}^{\infty}\rho^i = 0.25 \cdot \sum_{i=10}^{\infty} 0.75^i = \frac{0.25 \cdot 0.75^{10}}{1-0.75} =$
$= 0.75^{10} \approx 0.056.$

Answer: $p(L \geq 10) = 0.056.$