Question: Consider a random sample (WOR) of two households from a population of households having monthly income (in Rs.) as follows:

Household	1	2	3	4	5
Income (In Rs.)	1000	1200	900	1500	1300

Enumerate all possible samples (WOR) of size 2 and show that the sample mean gives an unbiased estimate of population mean.

Solution: The given population has total 5 households, and here we shall consider a random sample "without replacement". Therefore, there are total $\binom{5}{2} = 10$ possible outcomes for a sample of size 2.

In the following table, we represent these 10 outcomes and compute sample mean for each case:

Sample of size 2 (X_1, X_2)	Sample mean $\left(\overline{X} = \frac{X_1 + X_2}{2} \right)$
(1000, 1200)	$\frac{1000 + 1200}{2} = 1100$
(1000, 900)	$\frac{1000 + 900}{2} = 950$
(1000, 1500)	$\frac{1000 + 1500}{2} = 1250$
(1000, 1300)	$\frac{1000 + 1300}{2} = 1150$
(1200, 900)	$\frac{1200 + 900}{2} = 1050$
(1200, 1500)	$\frac{1200 + 1500}{2} = 1350$
(1200, 1300)	$\frac{1200 + 1300}{2} = 1250$
(900, 1500)	$\frac{900 + 1500}{2} = 1200$
(900, 1300)	$\frac{900 + 1300}{2} = 1100$
(1500, 1300)	$\frac{1500 + 1300}{2} = 1400$

True mean of the population is,

$$\mu = \frac{1000 + 1200 + 900 + 1500 + 1300}{5} = \frac{5900}{5} = 1180$$

As each of the 10 possible outcomes for the sample of size 2 is equally likely, each of them will occur with probability $\frac{1}{10}$. Therefore, the expectation is sample mean is,

$$E(\bar{X})$$
$$= \frac{1}{10} \times (1100 + 950 + 1250 + 1150 + 1050 + 1350 + 1250 + 1200 + 1100 + 1400)$$
$$= \frac{1}{10} \times 11800$$
$$= 1180 = \mu$$

Hence, we have, $E(\bar{X}) = \mu$

This shows that, **sample mean is unbiased estimate of population mean.** <u>***Answer***</u>