Answer to Question #83933 – Math – Trigonometry

Question

1. Express cos4tita in terms of powers of cos tita
2. Given that sin tita=-3/5 and cos alpha=-2/5. Find sin(A+B) if 1. tita and alpha are acute 2. tita is acute and alpha is obtuse.

Solution

1. We know that $\cos 2\theta = 2\cos^2\theta - 1$

$$\cos 4\theta = \cos 2(2\theta) = 2\cos^2 2\theta - 1$$
$$= 2(2\cos^2\theta - 1)^2 - 1$$
$$= 2(4\cos^4\theta - 4\cos^2\theta + 1) - 1$$
$$= 8\cos^4\theta - 8\cos^2\theta + 1$$

2. Given $\sin\theta = -\frac{3}{5} \Rightarrow \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(-\frac{3}{5}\right)^2} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

$$\cos\alpha = -\frac{2}{5} \Rightarrow \sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\left(-\frac{2}{5}\right)^2} = \sqrt{1-\frac{4}{25}} = \sqrt{\frac{21}{25}} = \frac{\sqrt{21}}{5}$$

CASE I : θ *and* α are acute then $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$ and $\cos\alpha = \frac{2}{5}$, $\sin\alpha = \frac{\sqrt{21}}{5}$

$$\sin(\theta+\alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha = \frac{3}{5}.\frac{2}{5} + \frac{4}{5}.\frac{\sqrt{21}}{5} = \frac{6+4\sqrt{21}}{25}$$

CASE II : θ is acute *and* α is obtuse then $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$ and $\cos\alpha = -\frac{2}{5}$, $\sin\alpha = \frac{\sqrt{21}}{5}$

$$\sin(\theta+\alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha = \frac{3}{5}.\left(-\frac{2}{5}\right) + \frac{4}{5}.\frac{\sqrt{21}}{5} = \frac{-6+4\sqrt{21}}{25}.$$