

## Answer to Question #83738, Math / Calculus

### Question

Find the inverse of the function  $f(x) = 6^{2x+7}$ . Give the domain of  $f^{-1}(x)$  in interval notation

### Solution

Given function is  $f(x) = 6^{2x+7}$

If a function  $f(x)$  is mapping  $x$  to  $y$ , then the inverse function of  $f(x)$  maps  $y$  back to  $x$ .

$$y = 6^{2x+7}$$

Interchange the variables  $x$  and  $y$

$$x = 6^{2y+7}$$

Solve  $x = 6^{2y+7}$  for  $y$

Taking log on both sides we get

$$\ln(x) = \ln(6^{2y+7})$$

$$\ln(x) = (2y + 7) \ln(6)$$

divide both sides by  $\ln(6)$

$$\frac{\ln(x)}{\ln(6)} = \frac{(2y + 7) \ln(6)}{\ln(6)}$$

$$\frac{\ln(x)}{\ln(6)} = 2y + 7$$

$$2y = \frac{\ln(x)}{\ln(6)} - 7$$

$$2y = \frac{\ln(x) - 7 \ln(6)}{\ln(6)}$$

$$y = \frac{\ln(x) - 7 \ln(6)}{2 \ln(6)}$$

Hence, required inverse of the given function is  $\frac{\ln(x) - 7 \ln(6)}{2 \ln(6)}$

Domain of  $f^{-1}(x)$ :

The domain of the function is the set of input or argument values for which the function is real and defined.

The function  $\frac{\ln(x)-7\ln(6)}{2\ln(6)}$  is defined for all  $x > 0$  and is real for  $x > 0$ .

Thus, the domain of the function is  $(0, \infty)$ .