Question

A shipment of ten similar plasma televisions contains 3 that are defective. If a hotel makes a random purchase of 2 of these television sets, find:
a) The probability distribution for the number of defectives television sets.
b) The probability that the hotel will purchase at least one defective television set.
c) The mean the defective television sets.
d) The variance of the defective television sets.

Solution

Let x be the number of defective sets purchased by the hotel.
Consider X is the random variable that represents the number of defective sets purchased by the hotel.
The total number of television sets is, $N = 10$.
The defective number of television sets is, $m = 3$.
The number of selected televisions for purchase is, $n = 2$.
The distribution of X follows hypergeometric distribution with parameters $N = 10, n = 2$ and $m = 3$.
a) The probability mass function for the hypergeometric distribution is as follows:

$$p(x, N, n, m) = Pr(X = x) = \frac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}}$$

$$Pr(X = x) = \frac{\binom{3}{x}\binom{10-3}{2-x}}{\binom{10}{2}}, x = 0, 1, 2$$

b) The probability that the hotel will purchase at least one defective television set

$$Pr(X \geq 1) = Pr(X = 1) + Pr(X = 2) = \frac{\binom{3}{1}\binom{10-3}{2-1}}{\binom{10}{2}} + \frac{\binom{3}{2}\binom{10-3}{2-2}}{\binom{10}{2}} =$$

$$= \frac{3(7) + \frac{3!}{2!\,(3-2)!}(1)}{\frac{10!}{2!\,(10-2)!}} = \frac{24}{45} = \frac{8}{15}$$

Check

$$Pr(X \geq 1) = 1 - Pr(X = 0) = 1 - \frac{\binom{3}{0}\binom{10-3}{2-0}}{\binom{10}{2}} = 1 - \frac{\frac{7!}{2!\,(7-2)!}(1)}{\frac{10!}{2!\,(10-2)!}} =$$

$$= 1 - \frac{21}{45} = \frac{8}{15}$$

c) The mean the defective television sets

$$\mu = E[X] = np = n\left(\frac{m}{N}\right) = 2\left(\frac{3}{10}\right) = \frac{3}{5}$$

d) The variance of the defective television sets.

$$\sigma^2 = Var[X] = \frac{np(1-p)(N-n)}{N-1} = \frac{n\left(\frac{m}{N}\right)\left(1-\frac{m}{N}\right)(N-n)}{N-1}$$

$$\sigma^2 = Var[X] = \frac{2\left(\frac{3}{10}\right)\left(1-\frac{3}{10}\right)(10-2)}{10-1} = \frac{28}{75}$$