The heights of 1000 students are approximately normally distributed with a mean of 154.5cm and a standard deviation of 3.0cm.

## Question

**a.** If a student is selected at random, what is the probability that his height is more than 158cm?

### Solution

$X \sim N(154.5, 3.0^2)$

$z = \dfrac{x - \mu}{\sigma}$

$z = \dfrac{158 - 154.5}{3} = 1.167$

$P(X > 158) = 1 - P(X \leq 158) = 1 - P(Z \leq 1.167) = 1 - 0.8783 = 0.1217$

## Question

**b.** How many of the students will be more than 158cm tall?

### Solution

$P(X > 158) = 0.1217$

$N = 1000$

$0.1217 \cdot 1000 \approx 122 \; students$

## Question

**c.** Obtain the probability that a randomly selected student has a height between 150cm and 160cm.

### Solution

$X \sim N(154.5, 3.0^2)$

$z = \dfrac{x - \mu}{\sigma}$

$z_1 = \dfrac{150 - 154.5}{3} = -1.500$

$z_2 = \dfrac{160 - 154.5}{3} = 1.833$

$P(150 < X < 160) = P(X < 160) - P(X \leq 150) =$

$$= P(Z < 1.833) - P(Z \leq -1.500) \approx 0.9666 - 0.0668 = 0.8998$$

## Question

**d.** What is the probability that a random sample of size 36 will have a mean height of more than 155cm?

## Solution

$$\mu_{\overline{X}} = \mu_X = 154.5$$

$$\sigma_{\overline{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{3.0}{\sqrt{36}} = 0.5$$

$$z = \frac{\overline{x} - \mu_{\overline{X}}}{\sigma_{\overline{X}}} = \frac{155 - 154.5}{0.5} = 1.000$$

$$P(\overline{X} > 155) = 1 - P(\overline{X} \leq 155) = 1 - P(Z \leq 1.000) = 1 - 0.8413 = 0.1587.$$