ANSWER on Question #82572 – Math – Calculus

QUESTION

Integrate

1.

$$x\ squared\ +\ x\ minus\ 1\ dx\ over\ 2-3x-\ x\ squared \to \int \frac{(x^2+x-1)dx}{2-3x-x^2}$$

2.

$$x\ cubed\ -\ 2x\ squared\ +\ 3x\ +4\ dx\ over\ x\ squared\ (\ x\ squared\ +\ 4x\ -12) \to$$

$$\int \frac{(x^3-2x^2+3x+4)dx}{x^2(x^2+4x-12)}$$

SOLUTION

1.

$$\int \frac{(x^2+x-1)dx}{2-3x-x^2}$$

Simplify the integrand

$$\frac{x^2+x-1}{2-3x-x^2}=\frac{(x^2+3x-2)-(2x+1)}{-(x^2+3x-2)}=\frac{(x^2+3x-2)}{-(x^2+3x-2)}+\frac{-(2x+1)}{-(x^2+3x-2)}=-1+\frac{2x+1}{x^2+3x-2}$$

$$\boxed{\frac{x^2+x-1}{2-3x-x^2}=-1+\frac{2x+1}{x^2+3x-2}}$$

$$x^2+3x-2=0 \to \begin{cases} a=1 \\ b=3 \\ c=-2 \end{cases} \to D=b^2-4ac=3^2-4\cdot 1\cdot(-2)=9+8=17>0 \to$$

$$\boxed{\left[\begin{array}{l} x_1=\dfrac{-3-\sqrt{17}}{2} \\ x_1=\dfrac{-3+\sqrt{17}}{2} \end{array}\right.} \to x^2+3x-2=\left(x-\left(\frac{-3-\sqrt{17}}{2}\right)\right)\left(x-\frac{-3+\sqrt{17}}{2}\right) \to$$

$$\boxed{x^2+3x-2=\left(\frac{2x+3+\sqrt{17}}{2}\right)\left(\frac{x+3-\sqrt{17}}{2}\right)}$$

Then,

$$\frac{2x+1}{x^2+3x-2}=\frac{2x+1}{\left(\frac{2x+3+\sqrt{17}}{2}\right)\left(\frac{x+3-\sqrt{17}}{2}\right)}=\frac{4\cdot(2x+1)}{\left(2x+3+\sqrt{17}\right)\left(2x+3-\sqrt{17}\right)}=$$

$$=\frac{A}{2x+3+\sqrt{17}}+\frac{B}{2x+3-\sqrt{17}}\rightarrow$$

$$\frac{4\cdot(2x+1)}{\left(2x+3+\sqrt{17}\right)\left(2x+3-\sqrt{17}\right)}=\frac{A}{2x+3+\sqrt{17}}+\frac{B}{2x+3-\sqrt{17}}\rightarrow$$

$$8x+4=A\left(2x+3-\sqrt{17}\right)+B\left(2x+3+\sqrt{17}\right)\rightarrow$$

$$8x+4=(2A+2B)x+\left(A\left(3-\sqrt{17}\right)+B\left(3+\sqrt{17}\right)\right)\rightarrow\begin{cases}2A+2B=8|\div(2)\\A\left(3-\sqrt{17}\right)+B\left(3+\sqrt{17}\right)=4\end{cases}\rightarrow$$

$$\begin{cases}A+B=4\rightarrow B=4-A\\A\left(3-\sqrt{17}\right)+(4-A)\left(3+\sqrt{17}\right)=4\end{cases}\rightarrow\begin{cases}B=4-A\\A\left(3-\sqrt{17}\right)+4\left(3+\sqrt{17}\right)-A\left(3+\sqrt{17}\right)=4\end{cases}\rightarrow$$

$$\begin{cases}B=4-A\\A\left(3-\sqrt{17}-3-\sqrt{17}\right)=4-4\left(3+\sqrt{17}\right)\end{cases}\rightarrow\begin{cases}B=4-A\\A\left(-2\sqrt{17}\right)=4-12-4\sqrt{17}\end{cases}\rightarrow$$

$$\begin{cases}B=4-A\\A\left(-2\sqrt{17}\right)=-8-4\sqrt{17}\end{cases}\rightarrow\begin{cases}B=4-A\\A\left(-2\sqrt{17}\right)=-4\left(2+\sqrt{17}\right)\Big|\div\left(-2\sqrt{17}\right)\end{cases}\rightarrow$$

$$\begin{cases}B=4-A\\A=\frac{-4\left(2+\sqrt{17}\right)}{-2\sqrt{17}}=\frac{2\left(2+\sqrt{17}\right)}{\sqrt{17}}\equiv\frac{2\left(17+2\sqrt{17}\right)}{17}\end{cases}\rightarrow\begin{cases}A=\frac{2\left(17+2\sqrt{17}\right)}{17}\\B=4-\frac{2\left(17+2\sqrt{17}\right)}{17}\end{cases}\rightarrow$$

$$\begin{cases}A=\frac{2\left(17+2\sqrt{17}\right)}{17}\\B=\frac{4\cdot17-2\left(17+2\sqrt{17}\right)}{17}\end{cases}\rightarrow\boxed{\begin{cases}A=\frac{2\left(17+2\sqrt{17}\right)}{17}\\B=\frac{2\left(17-2\sqrt{17}\right)}{17}\end{cases}}$$

Conclusion,

$$\frac{2x+1}{x^2+3x-2}=\frac{\frac{2\left(17+2\sqrt{17}\right)}{17}}{2x+3+\sqrt{17}}+\frac{\frac{2\left(17-2\sqrt{17}\right)}{17}}{2x+3-\sqrt{17}}\rightarrow$$

$$\frac{2x+1}{x^2+3x-2}=\frac{2}{17}\cdot\left(\frac{17+2\sqrt{17}}{2\cdot\left(x+\left(\frac{3+\sqrt{17}}{2}\right)\right)}+\frac{17-2\sqrt{17}}{2\cdot\left(x+\frac{3-\sqrt{17}}{2}\right)}\right)\rightarrow$$

$$\boxed{\frac{2x+1}{x^2+3x-2}=\frac{1}{17}\cdot\left(\frac{17+2\sqrt{17}}{\left(x+\left(\frac{3+\sqrt{17}}{2}\right)\right)}+\frac{17-2\sqrt{17}}{\left(x+\frac{3-\sqrt{17}}{2}\right)}\right)}$$

General conclusion,

$$\boxed{\frac{x^2+x-1}{2-3x-x^2}=-1+\frac{1}{17}\cdot\left(\frac{17+2\sqrt{17}}{\left(x+\left(\frac{3+\sqrt{17}}{2}\right)\right)}+\frac{17-2\sqrt{17}}{\left(x+\frac{3-\sqrt{17}}{2}\right)}\right)}$$

Then,

$$\int\frac{(x^2+x-1)dx}{2-3x-x^2}=\int\left(-1+\frac{1}{17}\cdot\left(\frac{17+2\sqrt{17}}{\left(x+\left(\frac{3+\sqrt{17}}{2}\right)\right)}+\frac{17-2\sqrt{17}}{\left(x+\frac{3-\sqrt{17}}{2}\right)}\right)\right)dx=$$

$$=-x+\frac{1}{17}\cdot\left(\left[17+2\sqrt{17}\right]\cdot\ln\left|x+\frac{3+\sqrt{17}}{2}\right|+\left[17-2\sqrt{17}\right]\cdot\ln\left|x+\frac{3-\sqrt{17}}{2}\right|\right)+Const$$

Thus,

$$\boxed{\int\frac{(x^2+x-1)dx}{2-3x-x^2}=-x+\frac{1}{17}\cdot\left(\left[17+2\sqrt{17}\right]\cdot\ln\left|x+\frac{3+\sqrt{17}}{2}\right|+\left[17-2\sqrt{17}\right]\cdot\ln\left|x+\frac{3-\sqrt{17}}{2}\right|\right)+Const}$$

2.

$$\int \frac{(x^3 - 2x^2 + 3x + 4)dx}{x^2(x^2 + 4x - 12)}$$

Simplify the integrand

$$\frac{(x^3 - 2x^2 + 3x + 4)}{x^2(x^2 + 4x - 12)} = \frac{(x^3 - 2x^2 + 3x + 4)}{x^2(x-2)(x+6)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-2)} + \frac{D}{(x+6)} \to$$

$$\frac{(x^3 - 2x^2 + 3x + 4)}{x^2(x-2)(x+6)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-2)} + \frac{D}{(x+6)} \Bigg| \times \left(x^2(x-2)(x+6)\right) \to$$

$$x^3 - 2x^2 + 3x + 4 = Ax(x-2)(x+6) + B(x-2)(x+6) + Cx^2(x+6) + Dx^2(x-2) \to$$

$$x^3 - 2x^2 + 3x + 4 = Ax(x^2 + 4x - 12) + B(x^2 + 4x - 12) + C(x^3 + 6x^2) + D(x^3 - 2x^2) \to$$

$$x^3 - 2x^2 + 3x + 4 = Ax^3 + 4Ax^2 - 12Ax + Bx^2 + 4Bx - 12B + Cx^3 + 6Cx^2 + Dx^3 - 2Dx^2 \to$$

$$x^3 - 2x^2 + 3x + 4 = (A + C + D)x^3 + (4A + B + 6C - 2D)x^2 + (-12A + 4B)x + (-12B) \to$$

$$\begin{cases} -12B = 4 \mid \div (-12) \\ -12A + 4B = 3 \\ 4A + B + 6C - 2D = -2 \\ A + C + D = 1 \end{cases} \to \begin{cases} B = -\dfrac{1}{3} \\ -12A - \dfrac{4}{3} = 3 \\ 4A - \dfrac{1}{3} + 6C - 2D = -2 \\ A + C + D = 1 \end{cases} \to \begin{cases} B = -\dfrac{1}{3} \\ -12A - \dfrac{4}{3} = 3 \\ 4A - \dfrac{1}{3} + 6C - 2D = -2 \\ A + C + D = 1 \end{cases} \to$$

$$\begin{cases} B = -\dfrac{1}{3} \\ -12A = \dfrac{4}{3} + 3 \\ 4A + 6C - 2D = -2 + \dfrac{1}{3} \\ A + C + D = 1 \end{cases} \to \begin{cases} B = -\dfrac{1}{3} \\ -12A = \dfrac{13}{3} \Big| \div (-12) \\ 4A + 6C - 2D = -\dfrac{5}{3} \\ A + C + D = 1 \end{cases} \to \begin{cases} B = -\dfrac{1}{3} \\ A = -\dfrac{13}{36} \\ 4 \cdot \left(-\dfrac{13}{36}\right) + 6C - 2D = -\dfrac{5}{3} \\ -\dfrac{13}{36} + C + D = 1 \end{cases} \to$$

$$\begin{cases} B = -\dfrac{1}{3} \\ A = -\dfrac{13}{36} \\ -\dfrac{13}{9} + 6C - 2D = -\dfrac{5}{3} \\ C + D = 1 + \dfrac{13}{36} \end{cases} \to \begin{cases} B = -\dfrac{1}{3} \\ A = -\dfrac{13}{36} \\ 6C - 2D = -\dfrac{15}{9} + \dfrac{13}{9} \\ C + D = \dfrac{49}{36} \end{cases} \to \begin{cases} B = -\dfrac{1}{3} \\ A = -\dfrac{13}{36} \\ 6C - 2D = -\dfrac{2}{9} \Big| \div (2) \\ C + D = \dfrac{49}{36} \end{cases} \to$$

$$\begin{cases} B = -\frac{1}{3} \\ A = -\frac{13}{36} \\ 3C - D = -\frac{1}{9} \\ C + D = \frac{49}{36} \end{cases} \to \begin{cases} B = -\frac{1}{3} \\ A = -\frac{13}{36} \\ C + D = \frac{49}{36} \\ C + D + 3C - D = -\frac{1}{9} + \frac{49}{36} \end{cases} \to \begin{cases} B = -\frac{1}{3} \\ A = -\frac{13}{36} \\ C + D = \frac{49}{36} \\ 4C = -\frac{4}{36} + \frac{49}{36} \end{cases} \to \begin{cases} B = -\frac{1}{3} \\ A = -\frac{13}{36} \\ C + D = \frac{49}{36} \\ 4C = \frac{45}{36} \Big| \div (4) \end{cases} \to$$

$$\begin{cases} B = -\frac{1}{3} \\ A = -\frac{13}{36} \\ \frac{45}{144} + D = \frac{49}{36} \\ C = \frac{45}{144} \end{cases} \to \begin{cases} B = -\frac{1 \cdot 48}{3 \cdot 48} \\ A = -\frac{13 \cdot 4}{36 \cdot 4} \\ D = \frac{49 \cdot 4}{144} - \frac{45}{144} \\ C = \frac{45}{144} \end{cases} \to \begin{cases} B = -\frac{48}{144} \\ A = -\frac{52}{144} \\ D = \frac{196 - 45}{144} \\ C = \frac{45}{144} \end{cases} \to \boxed{\begin{cases} A = -\frac{52}{144} \\ B = -\frac{48}{144} \\ C = \frac{45}{144} \\ D = \frac{151}{144} \end{cases}}$$

Conclusion,

$$\boxed{\frac{(x^3 - 2x^2 + 3x + 4)}{x^2(x^2 + 4x - 12)} = -\frac{52}{144} \cdot \frac{1}{x} - \frac{48}{144} \cdot \frac{1}{x^2} + \frac{45}{144} \cdot \frac{1}{x - 2} + \frac{151}{144} \cdot \frac{1}{x + 6}}$$

Then,

$$\int \frac{(x^3 - 2x^2 + 3x + 4)dx}{x^2(x^2 + 4x - 12)} = \int \left(-\frac{52}{144} \cdot \frac{1}{x} - \frac{48}{144} \cdot \frac{1}{x^2} + \frac{45}{144} \cdot \frac{1}{x - 2} + \frac{151}{144} \cdot \frac{1}{x + 6}\right) dx =$$

$$= -\frac{52}{144} \cdot \ln|x| - \frac{48}{144} \cdot \left(-\frac{1}{x}\right) + \frac{45}{144} \cdot \ln|x - 2| + \frac{151}{144} \cdot \ln|x + 6| + Const =$$

$$= \frac{45x \cdot \ln|x - 2| - 52x \cdot \ln|x| + 151x \cdot \ln|x + 6| + 48}{144x} + Const$$

Thus,

$$\boxed{\int \frac{(x^3 - 2x^2 + 3x + 4)dx}{x^2(x^2 + 4x - 12)} = \frac{45x \cdot \ln|x - 2| - 52x \cdot \ln|x| + 151x \cdot \ln|x + 6| + 48}{144x} + Const}$$

ANSWER:

1.

$$\int \frac{(x^2 + x - 1)dx}{2 - 3x - x^2}$$

$$= -x + \frac{1}{17} \cdot \left(\left[17 + 2\sqrt{17}\right] \cdot \ln\left|x + \frac{3 + \sqrt{17}}{2}\right| + \left[17 - 2\sqrt{17}\right] \cdot \ln\left|x + \frac{3 - \sqrt{17}}{2}\right| \right) + Const$$

2.

$$\int \frac{(x^3 - 2x^2 + 3x + 4)dx}{x^2(x^2 + 4x - 12)} = \frac{45x \cdot \ln|x - 2| - 52x \cdot \ln|x| + 151x \cdot \ln|x + 6| + 48}{144x} + Const$$