Answer on Question #82282 – Math – Differential Equations

Question

$$(x^3-1)y''(x)+(x^2)y'(x)+xy(x)=0, y(0)=2, y'(0)=1$$

Solution

$$y=\sum_{n=0}^{\infty} a_n x^n$$

$$y'=\sum_{n=1}^{\infty} a_n n x^{n-1}$$

$$y''=\sum_{n=2}^{\infty} a_n n(n-1)x^{n-2}$$

$$(x^3-1)\sum_{n=2}^{\infty} a_n n(n-1)x^{n-2}+(x^2)\sum_{n=1}^{\infty} a_n n x^{n-1}+x\sum_{n=0}^{\infty} a_n x^n=0$$

$$\sum_{n=2}^{\infty} a_n n(n-1)x^{n+1}-\sum_{n=2}^{\infty} a_n n(n-1)x^{n-2}+\sum_{n=1}^{\infty} a_n n x^{n+1}+\sum_{n=0}^{\infty} a_n x^{n+1}=0$$

$n-2\to N, n\to N+2$

$$\sum_{n=2}^{\infty} a_n n(n-1)x^{n+1}-\sum_{N+2=2}^{\infty} a_{N+2}(N+2)(N+2-1)x^{N+2-2}+$$

$$+\sum_{n=1}^{\infty} a_n n x^{n+1}+\sum_{n=0}^{\infty} a_n x^{n+1}=0$$

$n+1\to N, n\to N-1$

$$\sum_{N-1=2}^{\infty} a_{N-1}(N-1)(N-1-1)x^N-\sum_{N=0}^{\infty} a_{N+2}(N+2)(N+2-1)x^N+$$

$$+\sum_{N-1=1}^{\infty} a_{N-1}(N-1)x^N+\sum_{N-1=0}^{\infty} a_{N-1}x^N=0$$

$$\sum_{N=3}^{\infty} a_{N-1}(N-1)(N-1-1)x^N-a_2(2)(1)-a_3(3)(2)x-a_4(4)(3)x^2-$$

$$-\sum_{N=3}^{\infty} a_{N+2}(N+2)(N+1)x^N + a_1(1)x^2 + \sum_{N=3}^{\infty} a_{N-1}(N-1)x^N + a_0 x +$$

$$+a_1 x^2 + \sum_{N=3}^{\infty} a_{N-1}x^N = 0$$

$$\sum_{N=3}^{\infty}(a_{N-1}(N^2 - 3N + 2 + N - 1 + 1) - a_{N+2}(N^2 + 3N + 2))x^N - 2a_2 -$$

$$-6a_3 x - 12a_4 x^2 + a_1 x^2 + a_0 x + a_1 x^2 = 0$$

$$-2a_2 = 0$$

$$a_0 - 6a_3 = 0$$

$$-12a_4 + 2a_1 = 0$$

$$a_{N-1}(N^2 - 2N + 2) - a_{N+2}(N^2 + 3N + 2) = 0$$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y(0) = 2$$

$$a_0 = 2$$

$$y' = \sum_{n=1}^{\infty} a_n n x^{n-1}$$

$$y'(0) = 1$$

$$a_1 = 1$$

$$a_2 = 0$$

$$a_3 = \frac{1}{3}$$

$$a_4 = \frac{1}{6}$$

$$a_{N+2} = \frac{N^2 - 2N + 2}{N^2 + 3N + 2} \cdot a_{N-1}$$

$$y = 2 + x + \sum_{n=1}^{\infty} a_{n+2}x^{n+2}$$

$$a_{n+2} = \frac{n^2-2n+2}{n^2+3n+2} \cdot a_{n-1}, a_0 = 2.$$

Answer: $y = 2 + x + \sum_{n=1}^{\infty} a_{n+2}x^{n+2}$, $a_{n+2} = \frac{n^2-2n+2}{n^2+3n+2} \cdot a_{n-1}$, $a_0 = 2$.

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