

Answer on Question #82248 – Math – Calculus

Question

Siam wants to build a new garden. He wants to make a garden with perimeter P and area A and he also wants to keep the value of P^2/A as small as he can. What will be the lowest value of P^2/A ?

Solution

Assuming that his garden has rectangular shape, write an expression for P^2/A :

$$\frac{P^2}{A} = \frac{[2(a+b)]^2}{ab}$$

To find minimum, simply find derivative and make it equal to zero, suppose that a is variable. We'll have to use the *quotient rule*:

$$\begin{aligned} \left(\frac{P^2}{A}\right)' &= \left(\frac{[2(a+b)]^2}{ab}\right)' = 4 \left(\frac{a^2 + b^2 + 2ab}{ab}\right)' = 4 \frac{(a^2 + b^2 + 2ab)'ab - (ab)'(a^2 + b^2 + 2ab)}{(ab)^2} = \\ &= 4 \frac{2a^2b + 2ab^2 - a^2b - 2ab^2 - b^3}{a^2b^2} = 4 \frac{a^2 - b^2}{a^2b} = 0. \end{aligned}$$

We see that $a \neq 0, b \neq 0$, and we have two solutions:

$$a = -b$$

$$a = b$$

The first one doesn't fit since we suppose that Siam lives on the Earth. With the second solution we have

$$\frac{P^2}{A} = \frac{[2(a+a)]^2}{a^2} = 16.$$

Answer: $\min\left\{\frac{P^2}{A}\right\} = 16.$