Answer on Question #82233 – Math – Differential Equations

Question

Solve the initial value problem: y''−2y'+y=0; y(1)=y'(1)=0

A. y(0) = 3

B. y(0) = 1

C. y(0) = 2

D. y(0) = 0

Solution

This is a linear homogeneous second-order differential equation with constant coefficients. Solve the characteristic equation:

$$k^2 - 2k + 1 = 0$$

$$k_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{0}}{2} \rightarrow k_1 = k_2 = 1$$

The solutions of the characteristic equation are double roots $k_1 = k_2 = k$, then the general solution is then:

$$y(x) = C_1 e^{kx} + C_2 x e^{kx} = \{k = 1\} = C_1 e^x + C_2 x e^x$$

y(1)=y'(1)=0 initial conditions give the following system:

$$\begin{cases} C_1 e^x + C_2 x e^x = y(x) \\ C_1 e^x + C_2 e^x + C_2 x e^x = y'(x) \end{cases} \rightarrow \begin{cases} C_1 + C_2 = 0 \\ C_1 + 2C_2 = 0 \end{cases} \rightarrow \begin{cases} C_1 = 0 \\ C_2 = 0 \end{cases}$$

$$y(x) = 0$$

A.) y(0) = 3, y'(1)=0 initial conditions give the following system:

$$\begin{cases} C_1 e^x + C_2 x e^x = y(x) \\ C_1 e^x + C_2 e^x + C_2 x e^x = y'(x) \end{cases} \rightarrow \begin{cases} C_1 = 3 \\ 3e^1 + C_2 e^1 + C_2 1 e^1 = 0 \end{cases} \rightarrow \begin{cases} C_1 = 3 \\ C_2 = -3/2 \end{cases}$$

$$y(x) = \frac{3}{2}(2 - x)e^x$$

B.) y(0) = 1, y'(1)=0 initial conditions give the following system:

$$\begin{cases} C_1 e^x + C_2 x e^x = y(x) \\ C_1 e^x + C_2 e^x + C_2 x e^x = y'(x) \end{cases} \rightarrow \begin{cases} C_1 = 1 \\ e^1 + C_2 e^1 + C_2 1 e^1 = 0 \end{cases} \rightarrow \begin{cases} C_1 = 1 \\ C_2 = -1/2 \end{cases}$$

$$y(x) = \frac{1}{2}(2 - x)e^x$$

C.) y(0) = 2, y'(1)=0 initial conditions give the following system:

$$\begin{cases} C_1 e^x + C_2 x e^x = y(x) \\ C_1 e^x + C_2 e^x + C_2 x e^x = y'(x) \end{cases} \rightarrow \begin{cases} C_1 = 2 \\ 2e^1 + C_2 e^1 + C_2 1 e^1 = 0 \end{cases} \rightarrow \begin{cases} C_1 = 2 \\ C_2 = -1 \end{cases}$$

$$y(x) = (2 - x)e^x$$

D.) y(0) = 0, y'(1)=0 initial conditions give the following system:

$$\begin{cases} C_1 e^x + C_2 x e^x = y(x) \\ C_1 e^x + C_2 e^x + C_2 x e^x = y'(x) \end{cases} \rightarrow \begin{cases} C_1 = 0 \\ C_2 e^1 + C_2 1 e^1 = 0 \end{cases} \rightarrow \begin{cases} C_1 = 0 \\ C_2 = 0 \end{cases}$$

$$y(x) = 0$$

Answer:

solution of the IVP: $y(x) = 0$

A. $y(x) = \frac{3}{2}(2 - x)e^x$

B. $y(x) = \frac{1}{2}(2 - x)e^x$

C. $y(x) = (2 - x)e^x$

D. $y(x) = 0$

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