Answer on Question #82221 – Math – Differential Equations

Question

1.

$$2xyp + (x^2 + y^2)q = (x + y)z$$

Solution

$$\frac{dx}{2xy} = \frac{dy}{x^2 + y^2} = \frac{dz}{x + y}$$

$$\frac{dx + dy}{2xy + x^2 + y^2} = \frac{dz}{x + y}$$

$$\frac{d(x + y)}{(x + y)^2} = \frac{dz}{x + y}$$

$$\frac{d(x + y)}{x + y} = dz$$

$$z = \ln\big(c(x + y)\big)$$

Answer: $z = \ln\big(c(x + y)\big)$.

Question

2.

$$y(x + z)p + (z^2 - 2xz - x^2)q = y(x - z)$$

Solution

$$\frac{dx}{y(x + z)} = \frac{dy}{z^2 - 2xz - x^2} = \frac{dz}{y(x - z)}$$

$$\frac{dx}{x + z} = \frac{dz}{x - z}$$

Substitute:

$z = tx\,; z' = t'x + t$

$$t'x + t = \frac{x - tx}{x + tx}$$

$$(t'x+t)(x+tx)=x-tx$$

$$t'x^2+t'tx^2+tx+t^2x=x-tx$$

$$x(t+1)t'=1-2t-t^2$$

$$\int \frac{t+1}{1-2t-t^2}dt=\int \frac{dx}{x}$$

$$\frac{t+1}{1-2t-t^2}=-\frac{t+1}{\left(t+1+\sqrt{2}\right)\left(t+1-\sqrt{2}\right)}$$

$$\frac{t+1}{\left(t+1+\sqrt{2}\right)\left(t+1-\sqrt{2}\right)}=\frac{A}{t+1+\sqrt{2}}+\frac{B}{t+1-\sqrt{2}}$$

$$A\left(t+1-\sqrt{2}\right)+B\left(t+1+\sqrt{2}\right)=t+1$$

$$A+B=1$$

$$A\left(1-\sqrt{2}\right)+B\left(1+\sqrt{2}\right)=1$$

$$A=B=0.5$$

$$\int \frac{t+1}{1-2t-t^2}dt=-\frac{1}{2}\int \frac{dt}{t+1+\sqrt{2}}-\frac{1}{2}\int \frac{dt}{t+1-\sqrt{2}}=-\frac{1}{2}\ln\left|t+1+\sqrt{2}\right|-\frac{1}{2}\ln\left|t+1-\sqrt{2}\right|=$$

$$=\ln\left|\frac{1}{\sqrt{t^2+2t-1}}\right|$$

$$\ln\left|\frac{1}{\sqrt{t^2+2t-1}}\right|=\ln|cx|$$

$$\frac{1}{\sqrt{t^2+2t-1}}=cx$$

$$\frac{1}{\sqrt{\left(\frac{z}{x}\right)^2+2\frac{z}{x}-1}}=cx$$

$$\frac{x}{\sqrt{z^2+2zx-x^2}}=cx$$

$$z^2 + 2zx - x^2 = c_1$$

Now, choosing (x, y, z) as multipliers, we get:

$xdx + ydy + zdz = 0$

$x^2 + y^2 + z^2 = c_2$

$\phi(z^2 + 2zx - x^2, x^2 + y^2 + z^2) = 0$

Answer: $\phi(z^2 + 2zx - x^2, x^2 + y^2 + z^2) = 0$.