

Answer on Question #82198 – Math – Differential Equations

Question

$$(y + z)p - (z + x)q = x - y$$

Solution

Compare with $Pp + Qq = R$

$$P = y + z, Q = -(z + x), R = x - y$$

Consider Lagrange's auxiliary equations

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{y + z} = \frac{dy}{-(z + x)} = \frac{dz}{x - y} = \frac{dx + dy + dz}{y + z - z - x + x - y} = \frac{dx + dy + dz}{0}$$

$$dx + dy + dz = 0 \Rightarrow x + y + z = c_1$$

$$\frac{dx}{y + z} = \frac{dy}{-(z + x)} = \frac{dz}{x - y} = \frac{xdx + ydy - zdz}{xy + xz - yz - xy - xz + yz} = \frac{xdx + ydy - zdz}{0}$$

$$d(x^2 + y^2 - z^2) = 0 \Rightarrow x^2 + y^2 - z^2 = c_2$$

General solution is

$$F(x + y + z, x^2 + y^2 - z^2) = 0$$

Answer: $F(x + y + z, x^2 + y^2 - z^2) = 0$.