Answer on Question #82195 – Math – Calculus

Question

What is the maximum number of intersection points a hyperbola and a circle could have?

Solution

The canonical equation of circle is

$$x^2 + y^2 = R^2$$

The canonical equation of the hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The intersection points are determined by the solutions of the following system

$$\begin{cases} x^2 + y^2 = R^2 \\ \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1 \end{cases}$$

$$\begin{cases} x^2 + y^2 = R^2 \\ \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1 \end{cases} => \begin{cases} y^2 = R^2 - x^2 \\ \dfrac{x^2}{a^2} - \dfrac{R^2 - x^2}{b^2} = 1 \end{cases} => \begin{cases} y^2 = R^2 - x^2 \\ x^2\left(\dfrac{1}{a^2} + \dfrac{1}{b^2}\right) = 1 + \dfrac{R^2}{b^2} \end{cases}$$

$$=> \begin{cases} y^2 = R^2 - x^2 \\ x^2 = \dfrac{a^2(b^2 + R^2)}{a^2 + b^2} \end{cases} => \begin{cases} x^2 = \dfrac{a^2(b^2 + R^2)}{a^2 + b^2} \\ y^2 = \dfrac{b^2(R^2 - a^2)}{a^2 + b^2} \end{cases}$$

The first equation can have two different solutions. The second equation can have two different solutions.

Therefore, the maximum number of intersection points is 4.

Answer: 4.