Question

Obtain the solution of the equation.

xy'=4y.

Solution

$$xy' = 4y;$$

In this ODE we can do a separation of variables.

$$x\frac{dy}{dx} = 4y;$$

$$\frac{dy}{dx} = \frac{4y}{x};$$

$$\frac{dy}{y} = 4\frac{dx}{x};$$

$$\int \frac{dy}{y} = 4 \int \frac{dx}{x};$$

$$\ln y = 4 \ln x + A;$$

where A is a some constant. We can rewrite it:

$$A = 4 \ln B;$$

where B is another constant. Therefore, we have

$$\ln y = 4 \ln x + 4 \ln B = 4 \ln(Bx) = \ln(Bx)^4 = \ln(Cx^4);$$

where $C = B^4$.

Finally,

$$y(x) = Cx^4.$$

Answer: $y = Cx^4$.