Answer on Question #81595 – Math – Calculus

Question

The original function used to model the cost of producing x PortaBoys Game Systems was

$$C(x) = 80x + 150.$$

While developing their newest game, Sasquatch Attack!, the makers of the PortaBoy revised their cost function using a cubic polynomial. The new cost of producing x PortaBoys is given by

$$C(x) = 0.03x^3 - 4.5x^2 + 224x + 250.$$

Market research indicates that the demand function

$$p(x) = -1.5x + 250$$

remains unchanged. Find the production level x that maximizes the profit made by producing and selling x PortaBoys.

Solution

Total Cost: $TC = 0.03x^3 - 4.5x^2 + 224x + 250$

Total Revenue: $TR = p(x)x = (-1.5x + 250\,)x = -1.5x^2 + 250x$

Profit:$P(x) = TR - TC = -1.5x^2 + 250x - (0.03x^3 - 4.5x^2 + 224x + 250) =$
$= -0.03x^3 + 3x^2 + 26x - 250$

Find Marginal Revenue

$MR = \frac{d}{dx}(TR) = \frac{d}{dx}(-1.5x^2 + 250x\,) = -3x + 250$

Find Marginal Cost

$MC = \frac{d}{dx}(TC) = \frac{d}{dx}(0.03x^3 - 4.5x^2 + 224x + 250\,) = 0.09x^2 - 9x + 224$

Profit is maximized where $MR = MC$

$$-3x + 250 = 0.09x^2 - 9x + 224, x \geq 0$$

$$0.09x^2 - 6x - 26 = 0$$

$$9x^2 - 600x - 2600 = 0$$

$$x = \frac{600 \pm \sqrt{(-600)^2 - 4(9)(-2600)}}{2(9)} = \frac{100 \pm 30\sqrt{14}}{3}$$

Since $x \geq 0,$ we take $x = \frac{100 + 30\sqrt{14}}{3} \approx 70.75$

$P(70) = -0.03(70)^3 + 3(70)^2 + 26(70) - 250 = 5980$

$P(71) = -0.03(71)^3 + 3(71)^2 + 26(71) - 250 = 5981.67$

The maximal profit is reached when 71 PortaBoys Game Systems are produced and sold.