Answer on Question #80965 – Math – Differential Equations

Question

$$ydy + xdx = \sqrt{x^2 + y^2}\, dx$$

Solution

First let's rewrite the equation as

$$y\frac{dy}{dx} + x = \sqrt{x^2 + y^2}$$

$$\frac{y\dfrac{dy}{dx} + x}{\sqrt{x^2 + y^2}} = 1$$

Then we consider $U = \sqrt{x^2 + y^2}$ and differentiate it by x

$$\frac{dU}{dx} = (\sqrt{x^2 + y^2})'_x = \frac{y\dfrac{dy}{dx} + x}{\sqrt{x^2 + y^2}}$$

Finally we get

$$\frac{dU}{dx} = 1$$

$$dU = dx$$

$$U = \int dx = x + C$$

where C is a constant.
Now we replace U

$$\sqrt{x^2 + y^2} = x + C$$

$$y^2 = (x + C)^2 - x^2 = 2xC + C^2$$