

## Answer on Question #80842 – Math – Linear Algebra

### Question

Which of the following are binary operations on  $S = \{x \in \mathbb{R} | x > 0\}$ ? Justify your answer.

- i) The operation defined by  $x * y = |\ln(xy)|$  where  $\ln x$  is the natural logarithm.
- ii) The operation defined by  $x * y = x^2 + y^3$ .

### Solution

An operation is binary if the result of performing the operation on a pair of elements of  $S$  is again an element of  $S$ .

Here both operations i) and ii) require 2 arguments (marked as  $x$  and  $y$ ).

- i) If we take  $x = y = 1$ , then

$$x * y = |\ln(1 \cdot 1)| = 0.$$

In other words,  $x * y \notin S$  if  $x \in S, y \in S$ .

Therefore, the operation defined by  $x * y = |\ln(xy)|$  is not a binary operation.

- ii) The operation defined by  $x * y = x^2 + y^3$  is binary operation because  $x^2 + y^3 > 0$  if  $x > 0, y > 0$ . In other words,  $x * y \in S$  if  $x \in S, y \in S$ .

**Answer:** only ii) is a binary operation.