

Answer on Question #80787 – Math – Statistics and Probability

Question

An online retailer has two adverts posted in different parts of a well-known social networking website, Advertisement A and Advertisement B. An average of 2 ‘clicks’ are generated by Advertisement A during the period Monday 10.00 to 10.05am. There are on average 5 ‘clicks’ generated by Advertisement B during the same period. Calculate the probability that on a particular Monday between 10.00 and 10.05am:

- i. Advertisement A generates at most 3 clicks. (5 marks)
- ii. Advertisement A generates at least 4 clicks. (5 marks)
- iii. Advertisement B generates no more than 4 clicks. (5 marks)
- iv. Advertisement A generates exactly 2 clicks and Advertisement B exactly 2 clicks. (5 marks)
- v. At least 3 clicks are generated in total by the two advertisements.

Solution

Let X be a number of clicks on Advertisement A during the period Monday 10.00 to 10.05am. X is a random variable and has Poisson distribution with pdf:

$$P(X = k) = e^{-\lambda_1} \frac{\lambda_1^k}{k!}$$

where λ_1 is the average.

Let Y be a number of clicks on Advertisement B during the period Monday 10.00 to 10.05am. Y is a random variable and has Poisson distribution with pdf:

$$P(Y = k) = e^{-\lambda_2} \frac{\lambda_2^k}{k!}$$

where λ_2 is the average.

We have that X and Y are independent.

Let $X \sim \text{Poisson}(\lambda_1)$ and $Y \sim \text{Poisson}(\lambda_2)$. If X and Y are independent, then $X + Y \sim \text{Poisson}(\lambda_1 + \lambda_2)$.

i. The probability that Advertisement A generates at most 3 clicks

$$\begin{aligned} P(X \leq 3) &= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = \\ &= e^{-2} \frac{2^0}{0!} + e^{-2} \frac{2^1}{1!} + e^{-2} \frac{2^2}{2!} + e^{-2} \frac{2^3}{3!} = e^{-2} \left(1 + 2 + 2 + \frac{4}{3} \right) = \frac{19}{3} e^{-2} \approx \\ &\approx 0.8571 \end{aligned}$$

ii. The probability that Advertisement A generates at least 4 clicks

$$P(X \geq 4) = 1 - P(X \leq 3) =$$

$$\begin{aligned}
&= 1 - (P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)) = \\
&= 1 - \left(e^{-2} \frac{2^0}{0!} + e^{-2} \frac{2^1}{1!} + e^{-2} \frac{2^2}{2!} + e^{-2} \frac{2^3}{3!} \right) = 1 - \frac{19}{3} e^{-2} \approx 0.1429
\end{aligned}$$

iii. The probability that Advertisement B generates no more than 4 clicks

$$\begin{aligned}
P(Y \leq 4) &= P(Y = 0) + P(Y = 1) + P(Y = 2) + P(Y = 3) + P(Y = 4) = \\
&= e^{-5} \frac{5^0}{0!} + e^{-5} \frac{5^1}{1!} + e^{-5} \frac{5^2}{2!} + e^{-5} \frac{5^3}{3!} + e^{-5} \frac{5^4}{4!} = \\
&= e^{-5} \left(1 + 5 + \frac{25}{2} + \frac{125}{6} + \frac{625}{24} \right) = \frac{523}{8} e^{-5} \approx 0.4405
\end{aligned}$$

iv. The probability that Advertisement A generates exactly 2 clicks and Advertisement B exactly 2 clicks

$$P(X = 2 \& Y = 2) = P(X = 2) \cdot P(Y = 2) = e^{-2} \frac{2^2}{2!} \cdot e^{-5} \frac{5^2}{2!} = 25e^{-7} \approx 0.0228$$

v. The probability that at least 3 clicks are generated in total by the two advertisements

$$\begin{aligned}
P(X + Y \geq 3) &= 1 - P(X + Y \leq 2) = \\
&= 1 - (P(X + Y = 0) + P(X + Y = 1) + P(X + Y = 2)) = \\
&= 1 - \left(e^{-(2+5)} \frac{(2+5)^0}{0!} + e^{-(2+5)} \frac{(2+5)^1}{1!} + e^{-(2+5)} \frac{(2+5)^2}{2!} \right) = \\
&= 1 - e^{-7} \left(1 + 7 + \frac{49}{2} \right) = 1 - \frac{65}{2} e^{-7} \approx 0.9704.
\end{aligned}$$