Answer on Question #80755 – Math – Statistics and Probability

Question

An online retailer has two adverts posted in different parts of a well-known social networking website, Advertisement A and Advertisement B. An average of 2 'clicks' are generated by Advertisement A during the period Monday 10.00 to 10.05am. There are on average 5 'clicks' generated by Advertisement B during the same period. Calculate the probability that on a particular Monday between 10.00 and 10.05am:

i. Advertisement A generates at most 3 clicks. (5 marks)
ii. Advertisement A generates at least 4 clicks. (5 marks)
iii. Advertisement B generates no more than 4 clicks. (5 marks)
iv. Advertisement A generates exactly 2 clicks and Advertisement B exactly 2 clicks. (5 marks)
v. At least 3 clicks are generated in total by the two advertisements.

Solution

Let $X =$ the number of clicks on Advertisement A during the period Monday 10.00 to 10.05am. X is a random variable and has Poisson distribution with pdf:

$$P(X = k) = e^{-\lambda_1} \frac{{\lambda_1}^k}{k!}$$

where λ_1 is the average.

Let $Y =$ the number of clicks on Advertisement B during the period Monday 10.00 to 10.05am. Y is a random variable and has Poisson distribution with pdf:

$$P(Y = k) = e^{-\lambda_2} \frac{{\lambda_2}^k}{k!}$$

where λ_2 is the average.

Theorem: Let $X \sim Poisson(\lambda_1)$ and $Y \sim Poisson(\lambda_2)$. Also assume that X and Y are independent. Then $X + Y \sim Poisson(\lambda_1 + \lambda_2)$.

i. The probability that Advertisement A generates at most 3 clicks

$$P(X \le 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) =$$

$$= e^{-2}\frac{2^0}{0!} + e^{-2}\frac{2^1}{1!} + e^{-2}\frac{2^2}{2!} + e^{-2}\frac{2^3}{3!} = e^{-2}\left(1 + 2 + 2 + \frac{4}{3}\right) = \frac{19}{3}e^{-2} \approx$$

$$\approx 0.8571$$

ii. The probability that Advertisement A generates at least 4 clicks

$$P(X \ge 4) = 1 - P(X \le 3) =$$

$$= 1 - \big(P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)\big) =$$

$$= 1 - \left(e^{-2}\frac{2^0}{0!} + e^{-2}\frac{2^1}{1!} + e^{-2}\frac{2^2}{2!} + e^{-2}\frac{2^3}{3!} \right) = 1 - \frac{19}{3}e^{-2} \approx 0.1429$$

iii. The probability that Advertisement B generates no more than 4 clicks

$$P(Y \le 4) = P(Y = 0) + P(Y = 1) + P(Y = 2) + P(Y = 3) + P(Y = 4) =$$
$$= e^{-5}\frac{5^0}{0!} + e^{-5}\frac{5^1}{1!} + e^{-5}\frac{5^2}{2!} + e^{-5}\frac{5^3}{3!} + e^{-5}\frac{5^4}{4!} =$$
$$= e^{-5}\left(1 + 5 + \frac{25}{2} + \frac{125}{6} + \frac{625}{24}\right) = \frac{523}{8}e^{-5} \approx 0.4405$$

iv. Assume that X and Y are independent. Then the probability that Advertisement A generates exactly 2 clicks and Advertisement B exactly 2 clicks

$$P(X = 2 \,\&\, Y = 2) = P(X = 2) \cdot P(Y = 2) = e^{-2}\frac{2^2}{2!} \cdot e^{-5}\frac{5^2}{2!} = 25e^{-7} \approx 0.0228$$

v. The probability that at least 3 clicks are generated in total by the two advertisements

$$P(X + Y \ge 3) = 1 - P(X + Y \le 2) =$$
$$= 1 - \big(P(X + Y = 0) + P(X + Y = 1) + P(X + Y = 2)\big) =$$
$$= 1 - \left(e^{-(2+5)}\frac{(2 + 5)^0}{0!} + e^{-(2+5)}\frac{(2 + 5)^1}{1!} + e^{-(2+5)}\frac{(2 + 5)^2}{2!} \right) =$$
$$= 1 - e^{-7}\left(1 + 7 + \tfrac{49}{2}\right) = 1 - \tfrac{65}{2}e^{-7} \approx 0.9704.$$