Answer on Question #80621 – Math – Statistics and Probability

Question

An online retailer has two adverts posted in different parts of a well-known social networking website, Advertisement A and Advertisement B. An average of 2 'clicks' are generated by Advertisement A during the period Monday 10.00 to 10.05am. There are on average 5 'clicks' generated by Advertisement B during the same period. Calculate the probability that on a particular Monday between 10.00 and 10.05 am:

i. Advertisement A generates at most 3 clicks.
ii. Advertisement A generates at least 4 clicks.
iii. Advertisement B generates no more than 4 clicks.
iv. Advertisement A generates exactly 2 clicks and Advertisement B exactly 2 clicks.
v. At least 3 clicks are generated in total by the two advertisements.

Solution

The number of clicks is Poisson random variable. Its mean is its parameter λ. Then number of clicks generated by Advertisement A: $X_1 \sim Poiss(2)$, number of clicks generated by Advertisement B :$X_2 \sim Poiss(5)$.

(i)

$$P(X_1 \le 3) = P(X_1 = 0) + P(X_1 = 1) + P(X_1 = 2) + P(X_1 = 3) =$$
$$= \frac{2^0 e^{-2}}{0!} + \frac{2^1 e^{-2}}{1!} + \frac{2^2 e^{-2}}{2!} + \frac{2^3 e^{-2}}{3!} = \left(1 + 2 + 2 + \frac{4}{3}\right) e^{-2} = 0.857$$

(ii) $P(X_1 \ge 4) = 1 - P(X \le 3) = 1 - 0.857 = 0.143$

(iii)

$$P(X_2 \le 4) = P(X_2 = 0) + P(X_2 = 1) + P(X_2 = 2) + P(X_2 = 3) + P(X_2 = 4) =$$
$$= \frac{5^0 e^{-5}}{0!} + \frac{5^1 e^{-5}}{1!} + \frac{5^2 e^{-5}}{2!} + \frac{5^3 e^{-5}}{3!} + \frac{5^4 e^{-5}}{4!} = \left(1 + 5 + \frac{25}{2} + \frac{125}{6} + \frac{625}{24}\right) e^{-5} = 0.440$$

(iv)

$$P(X_1 = 2, X_2 = 2) = P(X_1 = 2)P(X_2 = 2) =$$
$$= \frac{2^2 e^{-2}}{2!} \cdot \frac{5^2 e^{-5}}{2!} = 25e^{-7} = 0.0228$$

(v)

$$P(X_1 + X_2 \ge 3) = 1 - P(X_1 + X_2 < 3) =$$
$$= 1 - \left(P(X_1 = 0, X_2 < 3) + P(X_1 = 1, X_2 < 2) + P(X_1 = 2, X_2 = 0)\right) =$$
$$= 1 - (P(X_1 = 0)\left(P(X_2 = 0) + P(X_2 = 1) + P(X_2 = 2)\right) +$$
$$+P(X_1 = 1)\left(P(X_2 = 0) + P(X_2 = 1)\right) + P(X_1 = 2)P(X_2 = 0)) =$$
$$= 1 - \left(e^{-2} \cdot e^{-5}\left(1 + 5 + \frac{5^2}{2}\right) + 2e^{-2} \cdot e^{-5}(1 + 5) + 2e^{-2} \cdot e^{-5}\right) = 0.970$$