Answer on Question #80519 – Math – Statistics and Probability

Question

A selection committee consisting of 10 members is to be formed from a group of 20 employees at a firm. Of the 20 employees, twelve are female. Calculate the probability that at most 8 committee members will be female.

Solution

Let A denote the event {at most 8 committee members are female}.

By the formula of probability of complementary event

$$P(A) = 1 - P(\bar{A}) = 1 - P(\{more\ than\ 8\ commitee\ members\ are\ female\})$$

The event $\bar{A}$ is the union of two mutually exclusive events: B - there are 9 female members and C there are 10 female members. Calculate probabilities of each of these events.

The total number of ways to choose a committee is $\binom{20}{10} = \frac{20!}{10!10!} = 184756$.

For the event B there are $\binom{12}{9} = \frac{12 \cdot 11 \cdot 10}{1 \cdot 2 \cdot 3} = 220$ ways to choose 9 female members and $\binom{8}{1} = 8$ ways to choose one male member. Then totally there are $220 \cdot 8 = 1760$ ways to choose a committee with 9 female members. Then

$$P(B) = \frac{1760}{184756}$$

For the event C there are $\binom{12}{10} = \frac{11 \cdot 12}{1 \cdot 2} = 66$ ways to choose 10 female members. Then

$$P(C) = \frac{66}{184756}$$

Then totally

$$P(\bar{A}) = P(B) + P(C) = \frac{1760 + 66}{184756} = 0.00988 \,,$$

$$P(A) = 1 - P(\bar{A}) = 1 - 0.00988 = 0.990 \,.$$