

Answer on Question #80474 – Math – Statistics and Probability

Question

The Average lifetime of a particular brand of car battery is 3 years with a standard deviation of 6 months. Assuming these lifetimes are normally distributed, calculate the:

- a) percentage of these batteries that last more than 4 years
- b) maximum lifetime of a battery to be in the 10% of batteries with the shortest lifetime

Solution

The lifetime of battery distribution is (in years)

$$X \sim N(3, 0.5^2).$$

Denote $F(x) = \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$. Values of F can be found in tables.

a) $P(X > 4) = P\left(\frac{X-3}{0.5} > \frac{4-3}{0.5}\right) = P(Z > 2) = F(2) = 0.0228$

So the percentage is 2.28%.

- b) Denote the desired lifetime by a . Then 10% of batteries have lifetime less than a . This means $P(X > a) = 0.9$.

We have

$$P(X > a) = P\left(\frac{X-3}{0.5} > \frac{a-3}{0.5}\right) = F(2(a-3)).$$

$$\text{So } F(2(a-3)) = 0.9$$

from which

$$2(a-3) = F^{-1}(0.9) = -1.28$$

and

$$a = 3 - \frac{1.28}{2} = 2.36$$

Converting 0.36 hour to minutes (multiplying by 60) gives the answer: 2 hours 22 minutes.