Answer on Question #80397 – Math – Calculus

Question

Find the length of the curve ,
$2y^2 = x^3$ from the vertex (0,0) to the point (4,4√2) .

Solution

For the top half of the curve we have

$$y = \frac{x^{3/2}}{\sqrt{2}} \quad ➔ \quad \frac{dy}{dx} = \frac{3}{2\sqrt{2}}x^{1/2}$$

$$y(0) = \frac{0^{3/2}}{\sqrt{2}} = 0 \quad \text{and} \quad y(4) = \frac{4^{3/2}}{\sqrt{2}} = \frac{8}{\sqrt{2}} = 4\sqrt{2} \text{ ➔ (0,0) and (4,}4\sqrt{2}$$

and so the arc length formula gives

$$L = \int_0^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_0^4 \sqrt{1 + \left(\frac{3}{2\sqrt{2}}x^{1/2}\right)^2}\, dx = \int_0^4 \sqrt{1 + \frac{9}{8}xdx} =$$

$$= \int_0^4 \sqrt{1 + \frac{9}{8}x}\, d\left(\frac{8}{9} \cdot \frac{9}{8}x\right) = \frac{8}{9} \cdot \int_0^4 \sqrt{1 + \frac{9}{8}xd}\left(\frac{9}{8}x\right) = \frac{8}{9} \cdot \int_0^4 \sqrt{1 + \frac{9}{8}xd}\left(1 + \frac{9}{8}x\right) =$$

If we substitute $u = 1 + \frac{9}{8}x$, then $du = \frac{9}{8}\, dx$. When $x = 0, u = 1$, when $x = 4, u = \frac{11}{2}$

$$L = \frac{8}{9}\int_1^{\frac{11}{2}} \sqrt{u}du = \frac{8}{9} \cdot \int_1^{\frac{11}{2}} u^{1/2}du = \frac{8}{9} \cdot \frac{1}{\frac{1}{2} + 1} \cdot u^{1/2+1}\Bigg]_1^{11/2} = \frac{8}{9} \cdot \frac{2}{3} \cdot u^{3/2}\Bigg]_1^{11/2} = \frac{16}{27} \cdot u^{3/2}\Bigg]_1^{11/2} =$$

$$= \frac{16}{27} \cdot \left[\left(\frac{11}{2}\right)^{3/2} - 1\right]$$

Without a substitution it will be

$$L = \frac{8}{9} \cdot \int_0^4 \sqrt{1 + \frac{9}{8}xd}\left(1 + \frac{9}{8}x\right) = \frac{8}{9} \cdot \int_0^4 \left(1 + \frac{9}{8}x\right)^{1/2} d\left(1 + \frac{9}{8}x\right) =$$

$$= \frac{8}{9} \cdot \frac{1}{\frac{1}{2} + 1} \cdot \left(1 + \frac{9}{8}x\right)^{1/2+1}\Bigg]_0^4 = \frac{8}{9} \cdot \frac{2}{3} \cdot \left(1 + \frac{9}{8}x\right)^{3/2}\Bigg]_0^4 = \frac{16}{27} \cdot \left[\left(1 + \frac{9}{8} \cdot 4\right)^{3/2} - \left(1 + \frac{9}{8} \cdot 0\right)^{3/2}\right] =$$

$$= \frac{16}{27} \cdot \left[\left(1 + \frac{9}{2}\right)^{3/2} - 1\right] = \frac{16}{27}\left[\left(\frac{11}{2}\right)^{3/2} - 1\right] = \frac{16}{27}\left[\frac{11}{2}\sqrt{\frac{11}{2}} - 1\right] = \frac{4}{27}\left[11\sqrt{11} \cdot \frac{4}{2\sqrt{2}} - 4\right] =$$

$$= \frac{4}{27}\left(11\sqrt{22} - 4\right) = \frac{1}{27}\left(44\sqrt{22} - 16\right) = \frac{44\sqrt{22} - 16}{27}$$

Answer: $\boldsymbol{L} = \frac{1}{27}\left(44\sqrt{22} - 16\right)$