Answer on Question #80394 – Math – Calculus

Question

Let a function f be defined as

$$f(x) = \begin{cases} a^2x, & if\ x > 1 \\ 5ax - 4, & if\ x \le 1 \end{cases}$$

Determine the value(s) of $'a'$ if any, for which f is continuous over R.

Solution

The function f is continuous on $(-\infty, 1) \cup (1, \infty)$ for $a \in R$.

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (5ax - 4) = 5a(1) - 4 = 5a - 4$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (a^2x) = a^2(1) = a^2$

We need to have

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$5a - 4 = a^2$

$a^2 - 5a + 4 = 0$

$(a - 1)(a - 4) = 0$

$a_1 = 1, a_2 = 4$

$a = 1$

$\lim_{x \to 1^-} f(x) = 5(1) - 4 = 1$

$\lim_{x \to 1^+} f(x) = (1)^2 = 1$

$\lim_{x \to 1^-} f(x) = 1 = \lim_{x \to 1^+} f(x) => \lim_{x \to 1} f(x) = 1$

$f(1) = 5(1)(1) - 4 = 1 = \lim_{x \to 1} f(x)$

Therefore, the function f is continuous at $x = 1$.

$a = 4$

$\lim_{x \to 1^-} f(x) = 5(4) - 4 = 16$

$\lim_{x \to 1^+} f(x) = (4)^2 = 16$

$\lim_{x \to 1^-} f(x) = 16 = \lim_{x \to 1^+} f(x) => \lim_{x \to 1} f(x) = 16$

$f(1) = 5(4)(1) - 4 = 16 = \lim_{x \to 1} f(x)$

Therefore, the function f is continuous at $x = 1$.

The function f is continuous over R, if $a = 1$ or $a = 4$.

Answer: $a = 1$ or $a = 4$.