Find the intervals in R over which integration (-1 to x) (t+1)3et dt is decreasing

Solution

Denote

$$f(x) = \int_{-1}^{x}(t+1)^3 e^t dt$$

f is decreasing if $f'(x) < 0$. We have (derivative of integral with respect to upper limit)

$$f'(x) = (x+1)^3 e^x$$

Then we need to solve an inequality

$(x+1)^3 e^x < 0$

Since e^x is always positive we have

$(x+1)^3 < 0$, or $x+1 < 0$, $x < -1$.

Answer: $(-\infty; -1)$