Answer on Question #80312 – Math – Algebra

Question

Find all the asymptotes of the curve $x^3 + y^3 = 3axy$, where $a > 0$

Solution

First we explore slant asymptotes of the curve. Let's write down the implicit equation in the form

$$x^3 + y^3 - 3axy = 0$$

Substituting the equation of the asymtote $y = kx + b$, we get

$$x^3 + (kx + b)^3 - 3ax(kx + b) = 0, \Rightarrow$$
$$\Rightarrow x^3 + k^3x^3 + 3k^2bx^2 + 3kb^2x + b^3 - 3akx^2 - 3abx = 0, \Rightarrow$$
$$\Rightarrow (1 + k^3)x^3 + (3k^2b - 3ak)x^2 + (3kb^2 - 3ab)x + b^3 = 0.$$

Equating coefficients at two senior terms to zero, we find the asymptote’s parameters of *k* and *b*:

$$\begin{cases} 1 + k^3 = 0, \\ 3k^2b - 3ak = 0 \end{cases} \Rightarrow \begin{cases} k^3 = -1, \\ 3k(kb - a) = 0 \end{cases} \Rightarrow k = -1 \Rightarrow -b - a = 0 \Rightarrow b = -a$$

Thus, the curve has the slant asymptote given by the equation

$$y = -x - a$$

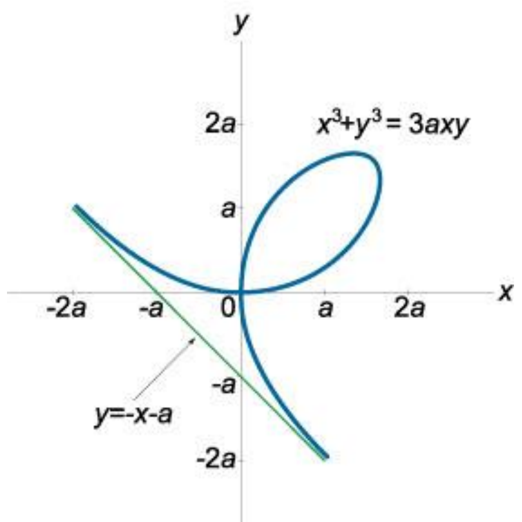

Figure 1

Let's check a possibility of existence of a vertical asymptote . We write its equation as $y = c$. Let's substitute it in the initial implicit equation of a curve:

$$x^3 + y^3 - 3axy = 0 \Rightarrow$$
$$\Rightarrow c^3 + y^3 - 3acy = 0, \Rightarrow y^3 - 3acy + c^3 = 0.$$

It means that the necessary condition of existence of a vertical asymptote is not satisfied. Therefore, the curve has only the slant asymptote found above.

Answer: the curve has the slant asymptote $y = -x - a$.