Answer on Question #80292 – Math – Calculus

Question

$$\int \frac{\cos(x)}{2\sin^2(x) - 3\sin(x) + 4} dx$$

Solution

$$\int \frac{\cos(x)}{2\sin^2(x) - 3\sin(x) + 4} dx$$

Substitution

$u = \sin(x), du = \cos(x)\, dx$

$$\int \frac{\cos(x)}{2\sin^2(x) - 3\sin(x) + 4} dx = \int \frac{1}{2u^2 - 3u + 4} du =$$

$$= \int \frac{1}{2\left(u^2 - 2\left(\frac{3}{4}\right)u + \left(\frac{3}{4}\right)^2\right) - 2\left(\frac{3}{4}\right)^2 + 4} du =$$

$$= \int \frac{1}{\frac{23}{8} + 2\left(u - \frac{3}{4}\right)^2} du = \frac{8}{23}\int \frac{1}{1 + \frac{16}{23}\left(u - \frac{3}{4}\right)^2} du$$

Substitution

$$v = \frac{4}{\sqrt{23}}\left(u - \frac{3}{4}\right), dv = \frac{4}{\sqrt{23}} du$$

$$\frac{8}{23}\int \frac{1}{1 + \frac{16}{23}\left(u - \frac{3}{4}\right)^2} du = \frac{2}{\sqrt{23}}\int \frac{1}{1 + v^2} dv = \frac{2}{\sqrt{23}} \arctan v + C =$$

$$= \frac{2}{\sqrt{23}} \arctan\left(\frac{4}{\sqrt{23}}\left(u - \frac{3}{4}\right)\right) + C$$

$$\int \frac{\cos(x)}{2\sin^2(x) - 3\sin(x) + 4} dx = \frac{2}{\sqrt{23}} \arctan\left(\frac{4}{\sqrt{23}}\left(u - \frac{3}{4}\right)\right) + C =$$

$$= \frac{2\sqrt{23}}{23} \arctan\left(\frac{4\sqrt{23}}{23}\left(\sin x - \frac{3}{4}\right)\right) + C$$