Answer on Question #80107 – Math – Calculus

Question

Does N satisfy Archimedean property ?

Solution

Let K be an ordered field. Since every ordered field is a linearly ordered group, all the above definitions of infinitely small and infinitely large elements, as well as the formulation of the axioms of Archimedes, remain valid. However, there are specific features that make the wording of the Archimedean axiom simpler.

Let a, b be positive elements of K.

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Infinitely small and infinitely large elements are united under the name of infinitesimal elements.

Accordingly, the wording of the axiom of Archimedes is simplified: the ordered field K has the property of Archimedes, if it contains no infinitesimal elements, or, equivalently, if there are no infinitely large elements. If here we expand the definition of an infinitely small (or infinitely large) element, then we obtain the following formulation of the axiom of Archimedes:

For every element a of the field K there exists a natural element n such that n> a.

If $0 < x < y$ are natural numbers, then there exists an $n \in \mathbb{N}$ such that $nx > y$.

(One can put $n = y + 1$).