

Question

$$k(x) = \frac{x^3 + 5x^2}{x^2 + 4x - 5}$$

limit of $k(x)$ as x approaches -5 from the right

limit of $k(x)$ as x approaches 1

limit of $k(x)$ as x approaches $-\infty$

Solution

limit of $k(x)$ as x approaches -5 from the right:

$$k(-5) = \frac{0}{0}$$

at -5 , the $k(x)$ has an uncertainty of the form $0/0$.

Solution version 1:

$$\lim_{x \rightarrow -5^+} \frac{x^3 + 5x^2}{x^2 + 4x - 5} = \lim_{x \rightarrow -5^+} \frac{x^2(x + 5)}{(x - 1)(x + 5)} = \lim_{x \rightarrow -5^+} \frac{x^2}{(x - 1)} = \frac{(-5)^2}{-5 - 1} = -\frac{25}{6}$$

Solution version 2: use L'Hospital's rule

$$\lim_{x \rightarrow -5^+} \frac{x^3 + 5x^2}{x^2 + 4x - 5} = \left\{ \lim_{x \rightarrow -5^+} k(x) = \lim_{x \rightarrow -5^+} k'(x) \right\} = \lim_{x \rightarrow -5^+} \frac{3x^2 + 10x}{2x + 4} = \frac{3(-5)^2 + 10(-5)}{2(-5) + 4} = -\frac{25}{6}$$

limit of $k(x)$ as x approaches 1 :

$$\lim_{x \rightarrow 1} \frac{x^3 + 5x^2}{x^2 + 4x - 5} = \frac{1^3 + 5 \cdot 1^2}{1^2 + 4 \cdot 1 - 5} = \frac{6}{0} = \infty$$

limit of $k(x)$ as x approaches $-\infty$:

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{x^3 + 5x^2}{x^2 + 4x - 5} &= \{ \text{divide by } x^3 \} = \lim_{x \rightarrow -\infty} \frac{1 + 5\frac{1}{x}}{\frac{1}{x} + 4\frac{1}{x^2} - 5\frac{1}{x^3}} = \{ \text{let } y = 1/x \} \\ &= \lim_{y \rightarrow 0^+} \frac{1 + 5y}{y + 4y^2 - 5y^3} = \frac{1 + 0}{0 + 4 \cdot 0^2 - 5 \cdot 0^3} = -\infty \end{aligned}$$

Answer:

$$\lim_{x \rightarrow -5^+} \frac{x^3 + 5x^2}{x^2 + 4x - 5} = -\frac{25}{6}, \quad \lim_{x \rightarrow 1} \frac{x^3 + 5x^2}{x^2 + 4x - 5} = \infty, \quad \lim_{x \rightarrow -\infty} \frac{x^3 + 5x^2}{x^2 + 4x - 5} = -\infty$$

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