Answer on Question #80051 – Math – Calculus

Question

integrate [(x+a)^-3(x+b)^-5]^1/4

Solution

The given integral is

$$\int [(x+a)^{-3}(x+b)^{-5}]^{\frac{1}{4}}\,dx.$$

Let's rewrite it as follows:

$$\int [(x+a)^{-3}(x+b)^{-5}]^{\frac{1}{4}}\,dx = \int \frac{dx}{\sqrt[4]{(x+a)^3(x+b)^5}} = \int \frac{dx}{\sqrt[4]{(x+a)^3(x+b)^{8-3}}} =$$

$$= \int \sqrt[4]{\left(\frac{x+b}{x+a}\right)^3}\,\frac{dx}{(x+b)^2} = \int \left(\frac{x+a}{x+b}\right)^{-\frac{3}{4}} \frac{dx}{(x+b)^2}.$$

By applying the differentiating rule for the quotient we get the formula:

$$\left(\frac{x+a}{x+b}\right)' = \frac{(x+a)'(x+b)-(x+a)(x+b)'}{(x+b)^2} = \frac{b-a}{(x+b)^2}.$$

Let's assume that

$$b-a \neq 0.$$

In that case

$$\frac{dx}{(x+b)^2} = \frac{1}{b-a}\,d\left(\frac{x+a}{x+b}\right).$$

So, using the substitution

$$\frac{x+a}{x+b} = z$$

we get:

$$\int \left(\frac{x+a}{x+b}\right)^{-\frac{3}{4}} \frac{dx}{(x+b)^2} = \int z^{-\frac{3}{4}} \frac{dz}{b-a} = \frac{1}{b-a}\cdot\frac{z^{-\frac{3}{4}+1}}{-\frac{3}{4}+1} + C = \frac{4}{b-a}\sqrt[4]{\frac{x+a}{x+b}} + C,$$

where C denotes an arbitrary integration constant.

In the remaining case

$$a = b$$

we have the integral

$$\int [(x+a)^{-3}(x+b)^{-5}]^{\frac{1}{4}}\, dx = \int (x+a)^{-2} dx.$$

Let's apply another substitution:

$$\int (x+a)^{-2} dx = [x+a=t] = \int t^{-2} dt = \frac{t^{-2+1}}{-2+1} + C = -\frac{1}{x+a} + C.$$

Answer:

$$\int [(x+a)^{-3}(x+b)^{-5}]^{\frac{1}{4}}\, dx = \begin{cases} \frac{4}{b-a}\sqrt[4]{\frac{x+a}{x+b}} + C, & a \neq b, \\ -\frac{1}{x+a} + C, & a = b. \end{cases}$$