Answer on Question #80047 – Math – Calculus

Question

$$I = \int \frac{e^x}{e^{\frac{x}{2}} - 1} dx$$

Solution

We can do a substitution:

$$e^{\frac{x}{2}} - 1 = t$$

Then

$$x = 2\ln(t+1);$$

$$t + 1 = e^{\frac{x}{2}};$$

$$dx = \frac{2}{t+1} dt \Rightarrow \frac{t+1}{2} dx = dt \Rightarrow e^{\frac{x}{2}} dx = 2dt;$$

$$e^x = e^{\frac{x}{2}} \cdot e^{\frac{x}{2}}.$$

So

$$I = \int \frac{(t+1)\cdot 2}{t} dt = 2 \cdot \int \frac{t+1}{t} dt = 2 \cdot \int \left(1 + \frac{1}{t}\right) dt = 2 \cdot \left(\int 1dt + \int \frac{1}{t} dt \right) =$$
$$= 2 \cdot \left(t + \ln(t) + const\right).$$

And finally put $t = e^{\frac{x}{2}} - 1$ back again:

$$I = 2 \cdot \left(e^{\frac{x}{2}} - 1 + \ln(e^{\frac{x}{2}} - 1) + const \right).$$

Answer: $\int \frac{e^x}{e^{\frac{x}{2}} - 1} dx = 2 \cdot \left(e^{\frac{x}{2}} - 1 + \ln(e^{\frac{x}{2}} - 1) + const \right).$