Answer on Question #79583 – Math – Calculus

Question

Form the PDE

2z=(ax+y)^2 + b

Solution

Given that

$$f(x, y, z) = 2z - (ax + y)^2 - b = 0 \qquad (1)$$

where z is a function of x and y, and a, b are arbitrary constants.

Form the PDE by eliminating arbitrary constants. Then differentiating (1) partially with respect to x and y respectively, we have

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial x} = -2a(ax + y) + 2\frac{\partial z}{\partial x} = 0 \quad \rightarrow z_x = a(ax + y) \quad (2)$$

$$\frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial y} = -2(ax + y) + 2\frac{\partial z}{\partial y} = 0 \quad \rightarrow z_y = ax + y \quad (3)$$

It follows from (2) and (3) that

$$\frac{z_x}{z_y} = \frac{a(ax + y)}{ax + y} = a \quad (4)$$

It follows from (3) and (4) that

$$z_y = \frac{z_x}{z_y}x + y$$

$$\left(\frac{\partial z}{\partial y}\right)^2 - x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0$$

Answer: $\left(\frac{\partial z}{\partial y}\right)^2 - x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0.$