Answer on Question #79305 – Math – Statistics and Probability

Question

Given the data

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

(a) Calculate the coefficient of correlation.
(b) Obtain the line of regression.
(c) Estimate the value of y which should correspond to $x = 6.2$

Solution

(a)

$$r = \frac{\sum(x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum(x_i - \overline{x})^2}\sqrt{\sum(y_i - \overline{y})^2}}$$

$$\overline{x} = \frac{1}{N}\sum x_i, \quad \overline{y} = \frac{1}{N}\sum y_i$$

$$\overline{x} = \frac{1}{9}(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) = 5$$

$$\overline{y} = \frac{1}{9}(9 + 8 + 10 + 12 + 11 + 13 + 14 + 16 + 15) = 12$$

$$\sum(x_i - \overline{x})(y_i - \overline{y}) = (1 - 5)(9 - 12) + (2 - 5)(8 - 12) +$$
$$+(3 - 5)(10 - 12) + (4 - 5)(12 - 12) + (5 - 5)(11 - 12) +$$
$$+(6 - 5)(13 - 12) + (7 - 5)(14 - 12) + (8 - 5)(16 - 12) +$$
$$+(9 - 5)(15 - 12) = 57$$

$$\sum(x_i - \overline{x})^2 = (1 - 5)^2 + (2 - 5)^2 + (3 - 5)^2 + (4 - 5)^2 + (5 - 5)^2 +$$
$$+(6 - 5)^2 + (7 - 5)^2 + (8 - 5)^2 + (9 - 5)^2 = 60$$

$$\sum(y_i - \overline{y})^2 = (9 - 12)^2 + (8 - 12)^2 + (10 - 12)^2 + (12 - 12)^2 +$$
$$+(11 - 12)^2 + (13 - 12)^2 + (14 - 12)^2 + (16 - 12)^2 + (15 - 12)^2 = 60$$

$$r = \frac{57}{\sqrt{60}\sqrt{\sum 60}} = 0.95$$

Strong correlation

(b) The line of regression

$$y = A + Bx$$

$$B = \frac{S_{xy}}{\sqrt{S_{xx}}\sqrt{S_{yy}}}, A = \overline{y} - B\overline{x}$$

$$S_{xx} = \frac{1}{N}\sum(x_i - \overline{x})^2$$

$$S_{yy} = \frac{1}{N}\sum(y_i - \overline{y})^2$$

$$S_{xy} = \frac{1}{N}\sum(x_i - \overline{x})(y_i - \overline{y})$$

$$S_{xx} = \frac{1}{9}(60) = \frac{20}{3}$$
$$S_{yy} = \frac{1}{9}(60) = \frac{20}{3}$$
$$S_{xy} = \frac{1}{9}(57) = \frac{19}{3}$$
$$B = \frac{\frac{19}{3}}{\sqrt{\frac{20}{3}}\sqrt{\frac{20}{3}}} = \frac{19}{20} = 0.95$$
$$A = 12 - 0.95(5) = 7.25$$
The line of regression
$$y = 7.25 + 0.95x$$

(c)
$$y = 7.25 + 0.95x$$
$$y(6.2) = 7.25 + 0.95(6.2) = 13.14$$
$$y(6.2) = 13.14$$

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