

Answer on Question #79236 – Math – Statistics and Probability

Question

Let $e=(i,j)$ represent an arbitrary resulting from two rolls of the four-sided die of example 2.1.1. Tabulate the discrete pdf and sketch the graph of the CDF for the

- (a) $Y(e) = i + j$
- (b) $Z(e) = i - j$
- (c) $W(e) = (i - j)^2$

Solution

The set of all possible outcomes is

$(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)$. Each of the outcomes has probability $1/16$.

- a) $Y(e) = i + j$

The set of possible values is $\{2,3,4,5,6,7,8\}$. The probabilities are:

$$P(Y = 2) = P((1,1)) = 1/16$$

$$P(Y = 3) = P((1,2)) + P((2,1)) = 2/16$$

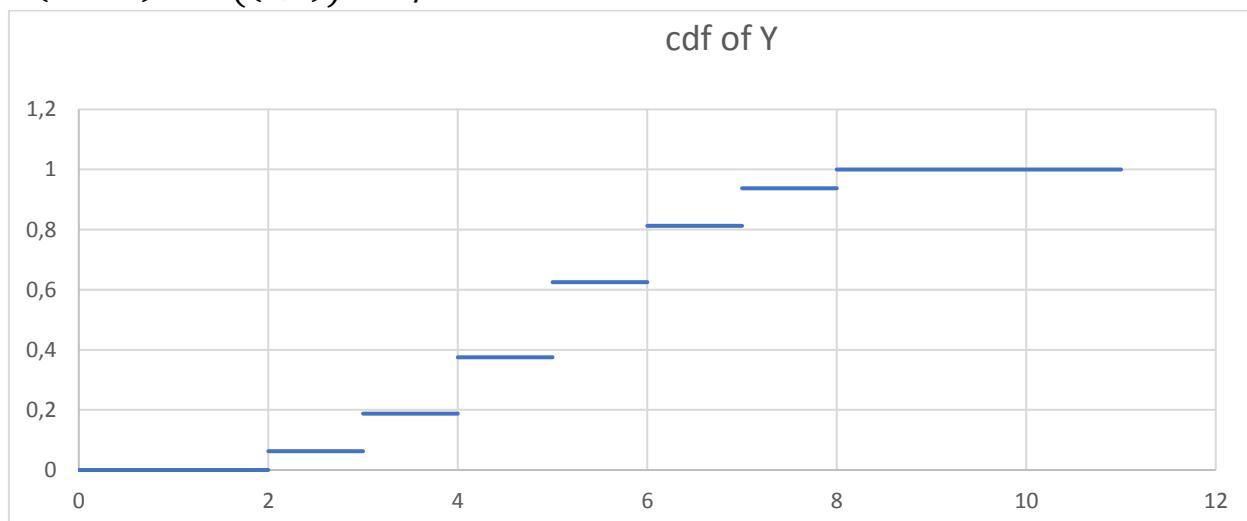
$$P(Y = 4) = P((1,3)) + P((2,2)) + P((3,1)) = 3/16$$

$$P(Y = 5) = P((1,4)) + P((2,3)) + P((3,2)) + P((4,1)) = 4/16$$

$$P(Y = 6) = P((2,4)) + P((3,3)) + P((4,2)) = 3/16$$

$$P(Y = 7) = P((3,4)) + P((4,3)) = 2/16$$

$$P(Y = 8) = P((4,4)) = 1/16$$

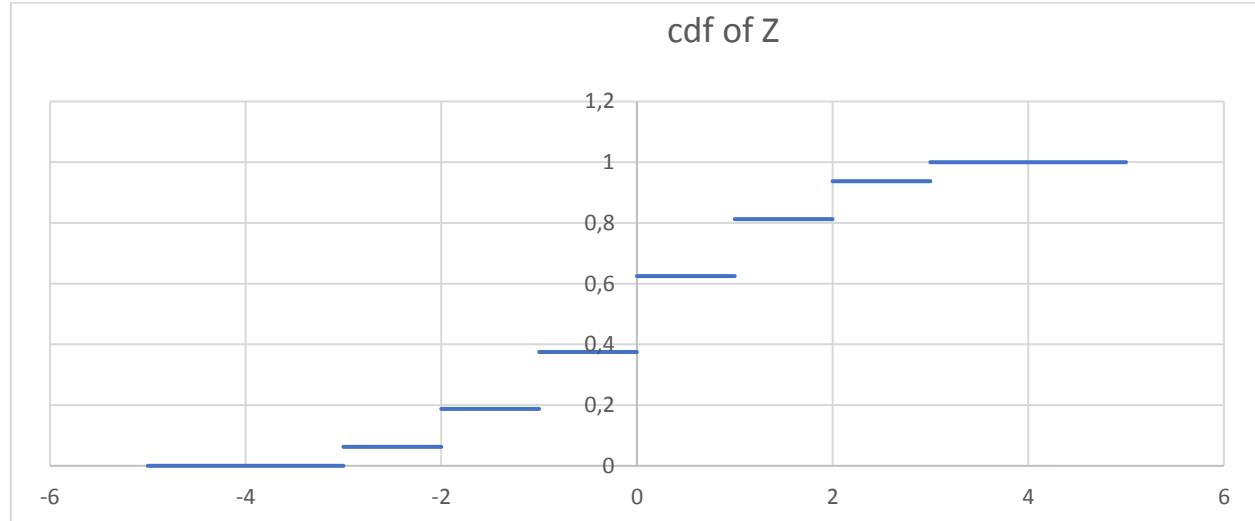


- b) $Z(e) = i - j$

The set of possible values is $\{-3, -2, -1, 0, 1, 2, 3\}$

$$P(Z = -3) = P((1,4)) = 1/16$$

$$\begin{aligned}
 P(Z = -2) &= P((1,3)) + P((2,4)) = 2/16 \\
 P(Z = -1) &= P((1,2)) + P((2,3)) + P((3,4)) = 3/16 \\
 P(Z = 0) &= P((1,1)) + P((2,2)) + P((3,3)) + P((4,4)) = 4/16 \\
 P(Z = 1) &= P((2,1)) + P((3,2)) + P((4,3)) = 3/16 \\
 P(Z = 2) &= P((3,1)) + P((4,2)) = 2/16 \\
 P(Z = 3) &= P((4,1)) = 1/16
 \end{aligned}$$



c) $W(e) = (i - j)^2$

The set of possible values is $\{0, 1, 4, 9\}$

$$\begin{aligned}
 P(Z = 0) &= P((1,1)) + P((2,2)) + P((3,3)) + P((4,4)) = 4/16 \\
 P(Z = 1) &= P((1,2)) + P((2,3)) + P((3,4)) + P((2,1)) + P((3,2)) + \\
 &\quad P((4,3)) = 6/16 \\
 P(Z = 4) &= P((1,3)) + P((2,4)) + P((3,1)) + P((4,2)) = 4/16 \\
 P(Z = 9) &= P((1,4)) + P((4,1)) = 2/16
 \end{aligned}$$

