ANSWER on Question #79130 – Math – Differential Equations

QUESTION

Find the value of n for which the equation

$$(n-1)^2 u_{xx} - y^{2n} u_{yy} = n y^{2n-1} u_y$$

is parabolic or hyperbolic.

SOLUTION

Consider the generic form of a second order linear partial differential equation in 2 variables with constant coefficients:

$$a u_{xx} + b u_{xy} + c u_{yy} + d u_x + e u_y + f u = g(x,y).$$

For the equation to be of second order, a, b, and c cannot all be zero($a^2 + b^2 + c^2 \neq 0$). Define its discriminant to be $D = b^2 - 4ac$. The properties and behavior of its solution are largely dependent of its type, as classified below.

If $D = b^2 - 4ac > 0$, then the equation is called hyperbolic.

If $D = b^2 - 4ac = 0$, then the equation is called parabolic.

If $D = b^2 - 4ac < 0$, then the equation is called elliptic.

(More information: https://en.wikipedia.org/wiki/Partial_differential_equation)

In our case,

$$(n-1)^2 u_{xx} - y^{2n} u_{yy} = n y^{2n-1} u_y \rightarrow (n-1)^2 u_{xx} - y^{2n} u_{yy} - n y^{2n-1} u_y = 0 \rightarrow \begin{cases} a = (n-1)^2 \\ b = 0 \\ c = -y^{2n} \\ d = 0 \\ e = -n y^{2n-1} \\ f = 0 \\ g(x,y) = 0 \end{cases}$$

$$D = b^2 - 4ac = 0^2 - 4 \cdot (n-1)^2 \cdot (-y^{2n}) = 4 \cdot (n-1)^2 \cdot (y^n)^2 = 0 \rightarrow n - 1 = 0 \rightarrow \boxed{n = 1}$$

Conclusion,

$$\boxed{\left[\begin{array}{l} if\ n = 1, D = 0 - equation\ is\ parabolic \\ if\ n \neq 1, D > 0 - equation\ is\ hyperbolic \end{array}\right.}$$

ANSWER: $\left[\begin{array}{l} if\ n = 1, D = 0 - equation\ is\ parabolic \\ if\ n \neq 1, D > 0 - equation\ is\ hyperbolic \end{array}\right.$