

Question

Solve the differential equations: $y=x(p^2-2p+2)$

Solution

$$\frac{d^2x}{dy^2} - 2\frac{dx}{dy} + 2x = y$$

CF:

$$\lambda^2 - 2\lambda + 2 = 0$$

$$\lambda_{1,2} = 1 \pm i$$

$$x_{CF} = (A \cos y + B \sin y)e^y$$

PI:

$$x_{PI} = Cy + D$$

$$0 - 2C + 2Cy + 2D = y$$

$$\begin{cases} 2C = 1 \\ 2D - 2C = 0 \end{cases} \rightarrow \begin{cases} C = 1/2 \\ D = 1/2 \end{cases}$$

$$x_{PI} = \frac{y+1}{2}$$

$$x = x_{CF} + x_{PI} = (A \cos y + B \sin y)e^y + \frac{y+1}{2}$$

Answer: $x = (A \cos y + B \sin y)e^y + \frac{y+1}{2}$