Answer on question #79058 – Math – Differential Equations

Question

Solve the initial value problem

$y' - 2xy = 1, y(0) = y_0$

y=e^−x2 (∫x0 e^−t2 dt+y0)
y=e^x2 (∫x0 e^−t2 dt+y0)
y=e^x2 (∫x0 e^t2 dt-y0)
y=(∫x0 e^−t2 dt+y0)

Solution

Let's rewrite the differential equation in the general form:

$$y' + p(x)y = g(x)$$

Multiplying both sides by some function $\mu = \mu(x)$, so that the left side is a total derivative. Hence,

$$\mu(y' + p(x)y) = (\mu y)' = \mu y' + \mu' y$$

From this equation we get

$$\mu p(x)y = \mu' y$$

$$\mu' = \mu p(x)$$

Integrating:

$$\mu(x) = e^{\int p(x)dx}$$

$$\left[e^{\int p(x)dx} y\right]' = e^{\int p(x)dx} g(x)$$

$$e^{\int p(x)dx} y = \int g(x) e^{\int p(x)dx} dx$$

Hence,

$$y = e^{-\int p(x)dx} \int g(x) e^{\int p(x)dx} dx$$

If having an IVP, we can rewrite the above equation as the following:

$$y = e^{-\int_{x_0}^{x} \int p(\tau)d\tau} \left[\int_{x_0}^{x} g(\tau) e^{\int p(\tau)d\tau} d\tau + y_0 \right]$$

So now we can use this general solution for our IVP, as $p(x) = -2x, g(x) = 1, x_0 = 0$. Hence,

$$y = e^{-\int_0^x \int -2\tau d\tau} \left[\int_0^x e^{\int -2\tau d\tau} d\tau + y_0 \right] = e^{x^2} \left[\int_0^x e^{-\tau^2} d\tau + y_0 \right]$$

So, the correct answer is the second one from the list.

Answer: $y = e^{x^2}\left[\int_0^x e^{-\tau^2} d\tau + y_0\right]$.