Answer on Question #79021 – Math – Calculus

Question

Verify f, defined by $f(x)=x^3+4x+1$ on [1,5] satisfies Lagrange's mean value theorem.

Solution

We should check that there exists $t \in (1,5)$ such that

$$f(5) - f(1) = f'(t) \cdot (5 - 1)$$

Calculate

$$\frac{f(5) - f(1)}{5 - 1} = \frac{125 + 20 + 1 - 1 - 4 - 1}{4} = 35$$

Then

$$f'(t) = 3t^2 + 4$$

$$3t^2 + 4 = 35$$
 for $t = \sqrt{31/3} \in (1,5)$

So

$$f(5) - f(1) = f'(\sqrt{31/3}) \cdot (5-1)$$

It means Lagrange's mean value theorem is satisfied.