Answer on Question #79019 – Math – Calculus

Question

Differentiate $\cos^{-1}(2x^2 - 1)$ with respect to $\sin^{-1}\sqrt{1-x^2}$.

Solution

$$f\left(\sin^{-1}\sqrt{1-x^2}\right) = \cos^{-1}(2x^2 - 1)$$

Differentiation of both sides gives us the following

$$f'\left(\sin^{-1}\sqrt{1-x^2}\right) = \left(\cos^{-1}(2x^2 - 1)\right)'$$

$$\frac{df\left(\sin^{-1}\sqrt{1-x^2}\right)}{d\sin^{-1}\sqrt{1-x^2}}\frac{d\sin^{-1}\sqrt{1-x^2}}{dx} = \left(\cos^{-1}(2x^2 - 1)\right)'$$

$$\frac{df\left(\sin^{-1}\sqrt{1-x^2}\right)}{d\sin^{-1}\sqrt{1-x^2}} = \left(\cos^{-1}(2x^2 - 1)\right)'\left(\left(\sin^{-1}\sqrt{1-x^2}\right)'\right)^{-1}$$

Since

$$\left(\cos^{-1}(2x^2 - 1)\right)' = -\frac{4x}{\sqrt{1-(2x^2-1)^2}}$$

and

$$\left(\sin^{-1}\sqrt{1-x^2}\right)' = \frac{-x}{|x|\sqrt{1-x^2}}$$

we obtain

$$\frac{df\left(\sin^{-1}\sqrt{1-x^2}\right)}{d\sin^{-1}\sqrt{1-x^2}} = \frac{4x}{\sqrt{1-(2x^2-1)^2}}\frac{|x|\sqrt{1-x^2}}{x} = 2$$

Answer: 2.