Answer on Question #79011 – Math – Differential Equations

Question

y′+2y=x^3e^−2x

y=e^−2x(x^4/4+c)

y=e^2x(x^4/4−c)

y=−e^−2x(x^4/4−c)

y=−e^2x(x^4/4+c)

Solution

The given equation

$$y' + 2y = x^3 e^{-2x} \quad (1)$$

is the ordinary linear differential equation, nonhomogeneous with constant coefficients. In order to solve it, one should obtain the general solution of the corresponding homogeneous equation and particular solution of the initial nonhomogeneous one.

In order to solve the homogeneous equation, one should obtain the characteristic polynomial (by substituting $y = e^{\lambda x}$):

$$\mathrm{y}' + 2\mathrm{y} = 0, \quad (2)$$

$$\lambda + 2 = 0, \quad \lambda = -2, \quad (3)$$

As a result, the general solution of (2) can be written in the following form:

$$y_0(x) = Ce^{-2x}. \quad (4)$$

The nonhomogeneous part of (1) is written in the form of a polynomial (of degree 3) multiplied by the exponent in power ax (with $a = -2$). As long as λ coincides with a (and we have only 1 root for it), one should select the following form for the particular solution:

$$y_1(x) = x^1 e^{-2x}(Ax^3 + Bx^2 + Cx + D) = e^{-2x}(Ax^4 + Bx^3 + Cx^2 + Dx). \quad (5)$$

Substituting (5) in (1), we come to the following expression:

$$-2e^{-2x}(Ax^4 + Bx^3 + Cx^2 + Dx) + e^{-2x}(4Ax^3 + 3Bx^2 + 2Cx + D) +$$

$$+2e^{-2x}(Ax^4 + Bx^3 + Cx^2 + Dx) = x^3 e^{-2x}. \quad (6)$$

Crossing off the equivalent terms and multipliers and comparing the coefficients in corresponding members in the left and right side of (6), we derive:

$$4A = 1, \quad A = \frac{1}{4},$$

$$\mathrm{B} = \mathrm{C} = \mathrm{D} = 0\,. \qquad (7)$$

Finally, the general solution of (1) can be written in the form as follows:

$$y(x) = y_0(x) + y_1(x) = Ce^{-2x} + e^{-2x}\frac{x^4}{4} = e^{-2x}\left(\frac{x^4}{4} + C\right), \qquad (8)$$

that coincides with the answer in line 1 of the question.

Answer: *y=e^(−2x)*(x^4/4+c).*