Answer on Question #78987 – Math – Calculus

Question

Differentiate tan^-1{(sinx-cosx)/(sinx+cosx)} with respect to x/2.

Solution

$$f(x) = \arctan\left\{\frac{\sin x - \cos x}{\sin x + \cos x}\right\}.$$

When we have a function $f(x) = f(g(x))$, then we can use the rule

$$\frac{df(x)}{dx} = \frac{df(g(x))}{dg(x)} \cdot \frac{dg(x)}{dx},$$

and then

$$\frac{df(g(x))}{dg(x)} = \frac{df(x)}{dx} \cdot \left(\frac{dg(x)}{dx}\right)^{-1}.$$

In our case we have a function $g(x) = x/2$, so

$$\frac{dg(x)}{dx} = \frac{d}{dx}\left(\frac{x}{2}\right) = \frac{1}{2},$$

$$\left(\frac{dg(x)}{dx}\right)^{-1} = 2.$$

Now we can find the derivative

$$\frac{df(x)}{dx} = \frac{d}{dx}\arctan\left\{\frac{\sin x - \cos x}{\sin x + \cos x}\right\} = \frac{1}{1 + \left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)^2} \cdot \left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)'$$

$$= \frac{1}{1 + \left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)^2} \cdot \frac{(\sin x - \cos x)'(\sin x + \cos x) - (\sin x - \cos x)(\sin x + \cos x)'}{(\sin x + \cos x)^2}$$

$$= \frac{(\cos x + \sin x)(\sin x + \cos x) - (\sin x - \cos x)(\cos x - \sin x)}{(\sin x + \cos x)^2 + (\sin x - \cos x)^2}$$

$$= \frac{(\sin x + \cos x)^2 + (\sin x - \cos x)^2}{(\sin x + \cos x)^2 + (\sin x - \cos x)^2} = 1.$$

Therefore, for our derivative we can write

$$\frac{df}{d\left(\frac{x}{2}\right)} = \frac{df(g(x))}{dg(x)} = 2 \cdot 1 = 2.$$

Answer: 2.