Answer on Question #78986 – Math – Calculus

Question

If $f(x) = \frac{4x^2 - 7x - 2}{x - 2}$, x is not equal to 0, find a $\delta > 0$ such that $|f(x) - 9| < \frac{1}{100}$ for

$0 < |x - 2| < \delta$. Hence show that $\lim_{x \to 2} f(x) = 9$.

Solution

Consider $|f(x) - 9| = \left|\frac{4x^2 - 7x - 2}{x - 2} - 9\right| = \left|\frac{4x^2 - 7x - 2 - 9(x - 2)}{x - 2}\right| = \left|\frac{4x^2 - 7x - 2 - 9x + 18}{x - 2}\right| = \left|\frac{4x^2 - 16x + 16}{x - 2}\right| = \left|\frac{4(x^2 - 4x + 4)}{x - 2}\right| = \left|\frac{4(x - 2)^2}{x - 2}\right| = |4(x - 2)| = 4|x - 2|.$

Since $|f(x) - 9| < \frac{1}{100}$ then $4|x - 2| < \frac{1}{100}$. Divide the last inequality by 4. Since $4 > 0$ then $|x - 2| < \frac{1}{400}$.

As δ one takes the number $\frac{1}{400}$.

Answer: $\delta = \frac{1}{400}$.