Answer on Question # 78968 – Math – Calculus

Question

Find the area enclosed by the curve r = a (1-cosθ).

Solution

Area = $2\int_0^{\pi}\frac{1}{2}r^2 d\theta$ .....(1),

where r = a (1-cosθ).

Now put the value of r in equation (1) and we get,

Area = $2\times\frac{1}{2}\int_0^{\pi}(a-a\,Cos\theta)^2 d\theta$

$= \int_0^{\pi}(a-a\,Cos\theta)^2\, d\theta$

$= \int_0^{\pi}(a^2-2a\,Cos\theta+a^2Cos^2\theta)\, d\theta$

$=\int_0^{\pi}\left(a^2-2a\,Cos\theta+\frac{1}{2}\times a^2\times 2Cos^2\theta\right)\, d\theta$

$= \int_0^{\pi}\left(a^2-2a\,Cos\theta+\frac{1}{2}\times a^2(1+Cos\,2\theta)\right)\, d\theta$

$= \int_0^{\pi} a^2\ d\theta - 2a\int_0^{\pi}(Cos\ \theta)\, d\theta\ +\frac{1}{2}\times a^2[\int_0^{\pi} d\theta + \int_0^{\pi} Cos\ 2\theta\, d\theta\ ]$

$= a^2\pi\ + 0 +\frac{1}{2}a^2[\pi+0]$ (As cos θ or cos 2θ integration over 0 to π is zero.)

$=\frac{3}{2}a^2\pi$.

Answer: Area enclosed by the curve is $\frac{3}{2}a^2\pi$.