Answer on Question #78964 – Math – Calculus
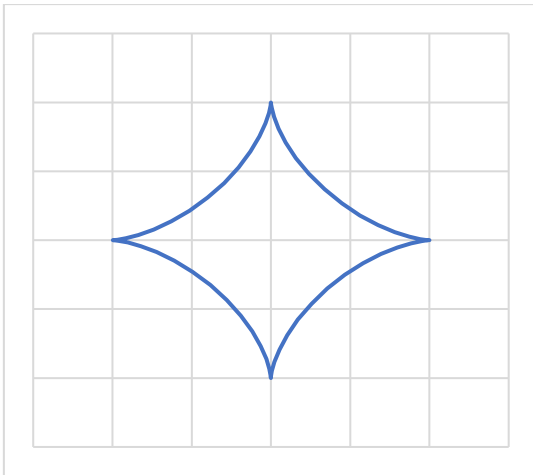
Question

Find the volume of the solid obtained by revolving the curve

x=a cos^3 theta

y=a sin^3 theta

about the y-axis.

Solution

Here is the graph

The formula is

$$V = \int_{-a}^{a} \pi x(y)^2 \, dy = [y = a\sin^3\theta, dy = 3a\sin^2\theta\cos\theta d\theta] = 3a \int_{-\pi/2}^{\pi/2} \pi(a\cos^3\theta)^2 \sin^2\theta\cos\theta d\theta =$$

$$= 3\pi a^3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^7\theta \sin^2\theta d\theta = 3\pi a^3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1 - \sin^2\theta)^3 \sin^2\theta d\sin\theta = [t = \sin\theta] = 3\pi a^3 \int_{-1}^{1}(1 - 3t^2 + 3t^4 - t^6)t^2 dt = 3\pi a^3 \left(\frac{2}{3} - 3*\frac{2}{5} + 3*\frac{2}{7} - \frac{2}{9}\right) = \frac{32}{105}\pi a^3$$

Answer: $\frac{32}{105}\pi a^3$.