Question

The function f defined by f(x)=x^3-6x^2+16x-15 is increasing in R. Is the statement true or false?

Give reason for your answer.

Solution

Let us find the derivative of a given function:

$$f'(x) = (x^3)' - (6x^2)' + (16x)' - (15)' = 3x^2 - 12x + 16$$

Critical points will occur when $f'(x) = 0$

$$\Downarrow$$

$$3x^2 - 12x + 16 = 0$$

$$D = 144 - 4 * 3 * 16 = 144 - 192 = -48$$

Since D < 0 there will be no solution to the equation.

Thus, the function will either be increasing or decreasing for all x.

Therefore, plug x = 0 into the derivative:

$$f'(0) = 3 * 0 - 12 * 0 + 16 = 16 - \text{it is positive}$$

$$\Downarrow$$

Thus, the function is increasing on all x.

Answer: the statement is true.