Answer on Question #78869 – Math – Calculus

Question

Trace the curve x(y^2+4)=8 stating all the points used for doing so.

Solution

$x(y^2+4)=8 \;\rightarrow\; x(y)=\frac{8}{y^2+4}$

$x(-y)=x(y)$ so the curve is symmetric over x-axis.

$x(0)=2$ so the curve does not pass through the origin.

Domain: $y=\pm\sqrt{\frac{8}{x}-4} \;\rightarrow\; 0<x<2.$

Range: $-\infty<y<\infty.$

Vertical asymptote: $x=0.$

x-intercept: (2,0).

y-intercept: no.

$\frac{d}{dx}\left(x(y^2+4)\right)=\frac{d}{dx}(8) \;\rightarrow\; y^2+4+2xy\frac{dy}{dx}=0 \;\rightarrow$

$\rightarrow\; \frac{dy}{dx}=-\frac{y^2+4}{2xy}=-\frac{4}{x^2y}.$

$For\ y>0,\ \frac{dy}{dx}<0 \rightarrow\ function\ decreases.$

$For\ y<0,\ \frac{dy}{dx}>0 \rightarrow\ function\ increases.$

$\frac{d^2y}{dx^2}=4\frac{2xy+x^2\frac{dy}{dx}}{x^4y^2}=8\frac{xy^2-2}{x^4y^2}.$

$\frac{d^2y}{dx^2}=0 \;\rightarrow\; xy^2-2=0 \rightarrow\; x\left(\frac{8}{x}-4\right)-2=0 \;\rightarrow\; x=\frac{3}{2}\ .$

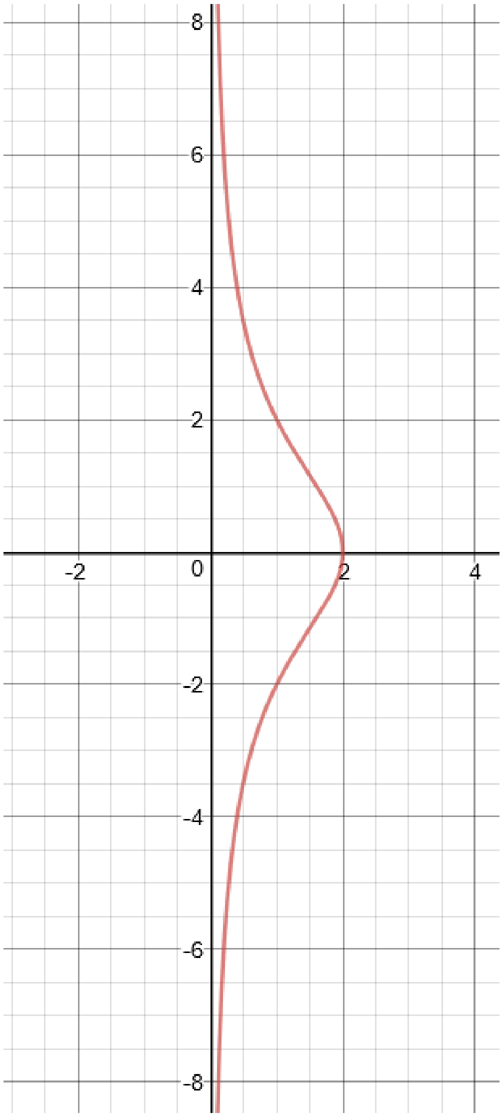
$$y\left(\frac{3}{2}\right) = \pm\sqrt{8 * \frac{2}{3} - 4} = \pm\frac{2}{\sqrt{3}}.$$

$$For\ 0 < x < \frac{3}{2},\ y < -\frac{2}{\sqrt{3}},\ \frac{d^2y}{dx^2} < 0\ \ \rightarrow\ \ the\ curve\ is\ concave\ down.$$

$$For\ \frac{3}{2} < x < 2,\ -\frac{2}{\sqrt{3}} < y < 0,\ \frac{d^2y}{dx^2} > 0\ \ \rightarrow\ \ the\ curve\ is\ concave\ up.$$

$$For\ \frac{3}{2} < x < 2,\ 0 < y < \frac{2}{\sqrt{3}},\ \frac{d^2y}{dx^2} < 0\ \ \rightarrow\ \ the\ curve\ is\ concave\ down.$$

$$For\ 0 < x < \frac{3}{2},\ y > \frac{2}{\sqrt{3}},\ \frac{d^2y}{dx^2} > 0\ \ \rightarrow\ \ the\ curve\ is\ concave\ up.$$



8
6
4
2
0
-2
2
4
-2
-4
-6
-8