

Answer on Question #78867 – Math – Calculus

Question

For which value(s) of k , is the function f , defined as below, continuous at $x = 2$?

$f(x) = \begin{cases} 3 - kx, & 1 \leq x < 2 \\ \end{cases}$

$\begin{cases} \frac{x^2}{4} - 3, & x \geq 2 \end{cases}$

Further, at which other points in $[1, \infty[$ is continuous, and why?

Solution

Function $f(x)$ is continuous at $x = 2$ if:

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) \rightarrow \lim_{x \rightarrow 2^-} (3 - kx) = \lim_{x \rightarrow 2^+} \left(\frac{x^2}{4} - 3 \right) \rightarrow$$
$$\rightarrow 3 - 2k = \frac{4}{4} - 3 \rightarrow k = \frac{5}{2}.$$

Functions $3 - kx$ and $\frac{x^2}{4} - 3$ are continuous on $(-\infty, \infty)$, so $f(x)$ is continuous on $[1, \infty)$.